

# **Theory of Cross-Phase Evolution and its Impact on ELM Dynamics**

**P.H. Diamond**

**CMTFO and CASS, UCSD, USA**

**WCI Center for Fusion Theory, NFRI, Korea**

**2014 US-TTF**

# **A Different Look at ELM Dynamics**

## **→ Thoughts on Selected Issues**

**P.H. Diamond**

**CMTFO and CASS, UCSD, USA**

**WCI Center for Fusion Theory, NFRI, Korea**

**KITP, UCSB, USA**

**2014 US-TTF**

# Collaborators

- Pengwei Xi<sup>(1,2)</sup>, X.-Q. Xu<sup>(2)</sup>

on cross phase correlation, anomalous dissipation

- T. Rhee<sup>(3)</sup>, J.M. Kwon<sup>(3)</sup>, W.W. Xiao<sup>(3,4)</sup>

on reduced models

- R. Singh<sup>(3)</sup>

on anomalous dissipation

1) Peking University

2) LLNL

3) NFRI

4) UCSD

# Acknowledgements

---

- Weiwon Xiao, Xiao-Lan Zou, Todd Evans, Rajesh Mainji, Sang-Hee Hahn, Michael Leconte, Guilhem Dif-Pradalier
- Utilized materials from APS Review by Tony Leonard

# Caveat Emptor

---

- Not a professional ELM-ologist
- Perspective is theoretical, and focus is on issues in **understanding dynamics**
- Perspective is that of a transport theorist
- Aim is to distill elements critical to model building
- Unresolved issues are discussed
- Not a review!

N.B. : Many ideas discussed here are  
contrary to ‘conventional wisdom’  
of ELM-ology

↔ Locale has  
a history of  
struggle against  
group think...



# Outline

---

- The conventional wisdom of ELMs
  - Motivation
  - Mechanism
- Some issues in ELM dynamics
  - How do bursts occur?
  - Mechanism of anomalous dissipation?
  - Assembling the ‘big picture’ → sources and transport effects?

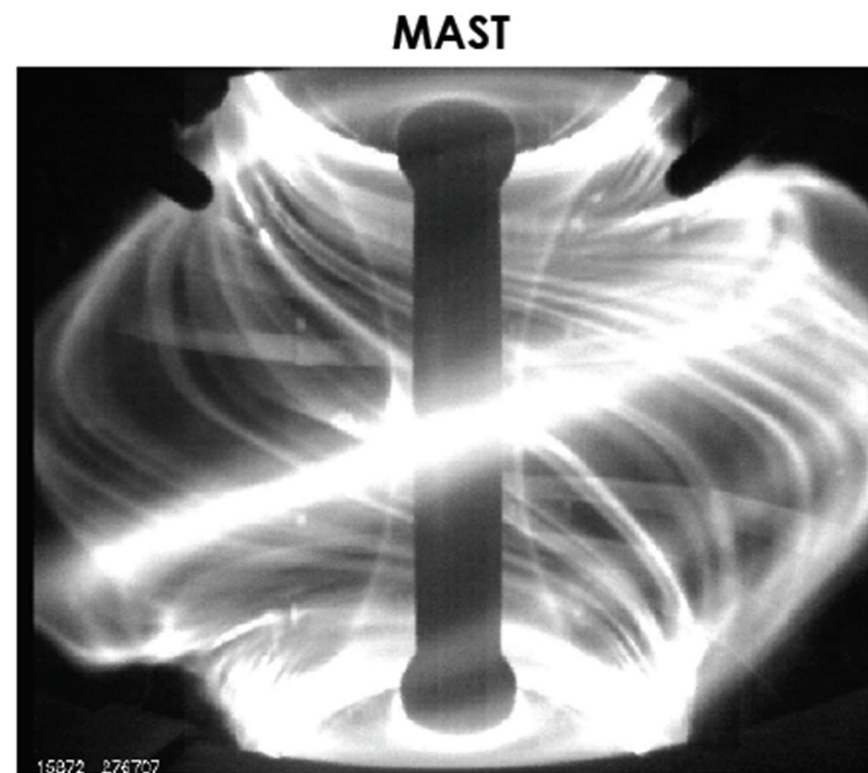
# Outline, cont'd

- Recent progress on some issues:
  - i) cross phase coherence and the origin of bursts
  - ii) phase coherence as leverage for ELM mitigation
  - iii) hyper-resistivity: single scale or multi-scale!?
  - iv) a reduced model of the big picture → importance of flux-drive
- Conclusions – at this point
- Discussion: where should we go next?



# Terra Firma: Conventional Wisdom of ELMs

- ELMs are ~ quasi-periodic relaxation events occurring at edge pedestal in H-mode plasma
- ELMs
  - Limit edge pedestal –
  - Expel impurities +
  - Damage PFC –
- ELMs → a serious concern for ITER
  - $\Delta W_{ELM} \sim 20\% W_{ped} \sim 20 \text{ MJ}$
  - $W_{ELM} / A \sim 10 \times \text{limit for melting}$
  - $\tau_{rise} \sim 200 \mu\text{sec}$



# Terra Firma: Conventional Wisdom of ELMs

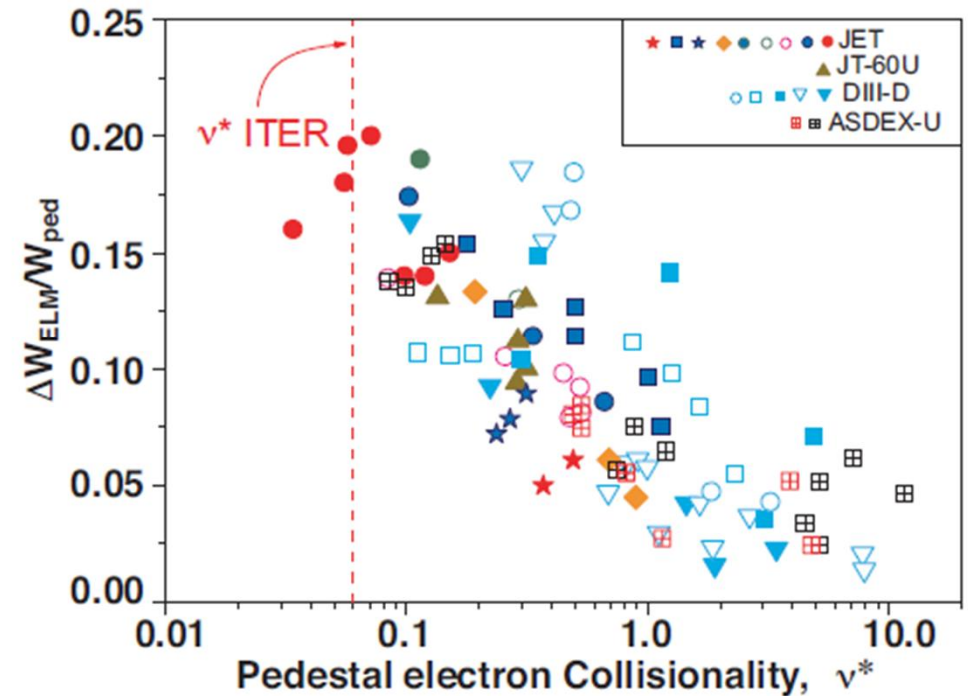
- ELM Types

- I, II:  $\omega_{ELM} \uparrow$  as  $P \uparrow$ , greatest concern, related to ideal stability
- III:  $\omega_{ELM} \downarrow$  as  $P \uparrow$ , closer to  $P_{Th}$ , unknown  $\rightarrow$  resistive ??

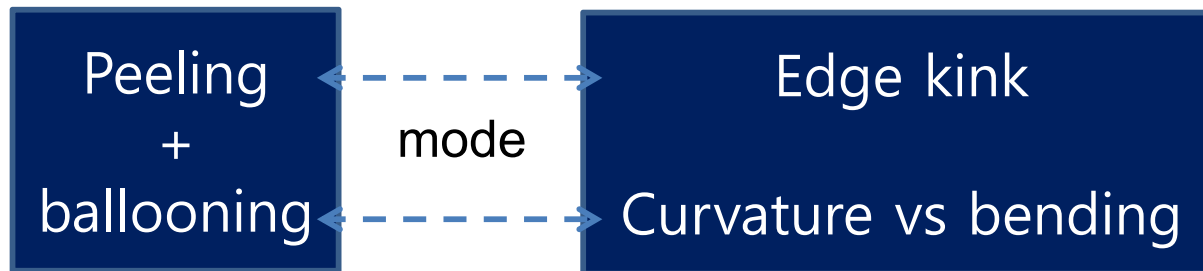
- Physics

- Type I, II ELM onset  $\rightarrow$  ideal stability limit
- i.e. peeling + ballooning

Relative ELM Energy Loss



[A. Loarte, *Plasma Phys. Control. Fusion* 45 1549 (2003)]



$\delta W$   
+  
Pedestal, geometry

# Terra Firma: Conventional Wisdom of ELMs

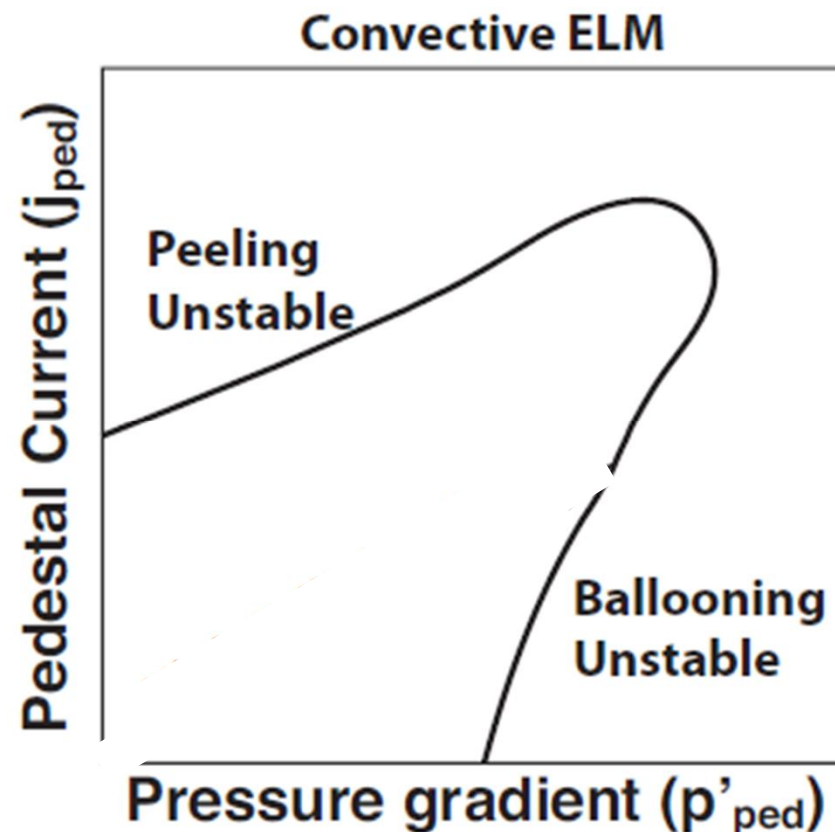
- Edge pressure gradient is ultimate energy source

$$- \delta W_P \sim \frac{1}{R_c} \frac{dP}{dr} \xi^2 \text{ vs } \delta W_{LB}$$

↔ ballooning

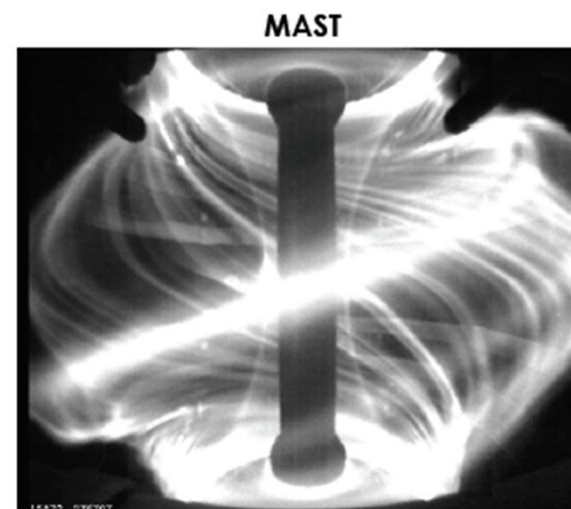
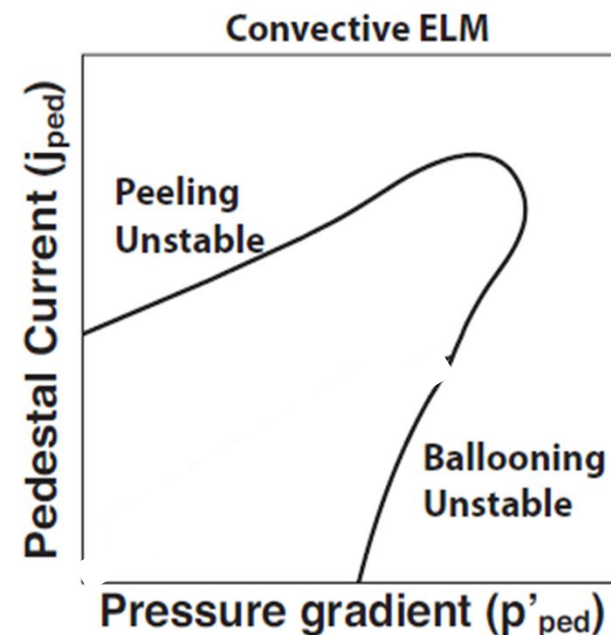
$$- J_{bootstrap} \sim \frac{1}{B_\theta (1 + 0.9\sqrt{v_*})} \frac{dP}{dr}$$

↔ peeling



# Terra Firma: Conventional Wisdom of ELMs

- Some relation of ELM drive character to collisionality is observed
  - Low collisionality  $\rightarrow$  peeling  $\sim$  more conductive
  - High collisionality  $\rightarrow$  ballooning  $\sim$  more convective
- Pedestal perturbation structure resembles P-B eigen-function structure
- Many basic features of ELMs consistent with ideal MHD peeling-ballooning theory



# Some Physics Questions

- What IS the ELM?

- ELMs single helicity or multi-helicity phenomena?

Relaxation event  $\leftrightarrow$  pedestal avalanche?

- How and why do actual **bursts** occur?

Why doesn't turbulence force  $\nabla P \sim \nabla P_{crit}$  oscillations?

- Pedestal turbulence develops during ELM. Thus, how do P-B modes interact with turbulence? – either ambient or as part of MH interaction?

- Does, or even should, the linear instability boundary define the actual ELM threshold?

# Some Physics Questions

- Irreversibility?

- Peeling-Ballooning are **ideal** modes. What is origin of irreversibility? How does fast reconnection occur?
- If hyper-resistivity is the answer (Xu et al, 2010), what is its origin – ambient micro-turbulence or P-B's themselves? Can P-B modes drive the requisite hyper-resistivity?
- What is the relation between hyper-resistivity, reconnection and heat transport, especially for 'conductive' ELMs?

# Some Physics Questions

- How do the pieces fit together?
  - Do ELM events emerge from a model which evolves profiles with pedestal turbulence?
  - What profiles are actually achieved at the point of ELMs?
  - What is the minimal model in which ELMs emerge?
  - What are the necessary ingredients in a full model?
- How exploit **dynamical** insight for ELM mitigation?

# I) Basic Notions of ELMs:

## ELM Bursts and Thresholds as

## Consequence of Stochastic Phase Dynamics

→ See P.W. Xi, X.-Q. Xu, P.D.; PRL 2014  
P.W. Xi, X.-Q. Xu, P.D.; PoP 2014 in press



# Simulation model and equilibrium in BOUT++

- 3-field model for nonlinear ELM simulations
  - ✓ Including essential physics for the onset of ELMs

- Peeling-ballooning instability
- Resistivity
- Hyper-resistivity
- Ion diamagnetic effect

$$\frac{d\varpi}{dt} = B \nabla_{\parallel} J_{\parallel} + 2\mathbf{b}_0 \times \boldsymbol{\kappa} \cdot \nabla \tilde{P} + \mu_{i,\parallel} \partial_{\parallel}^2 \varpi$$

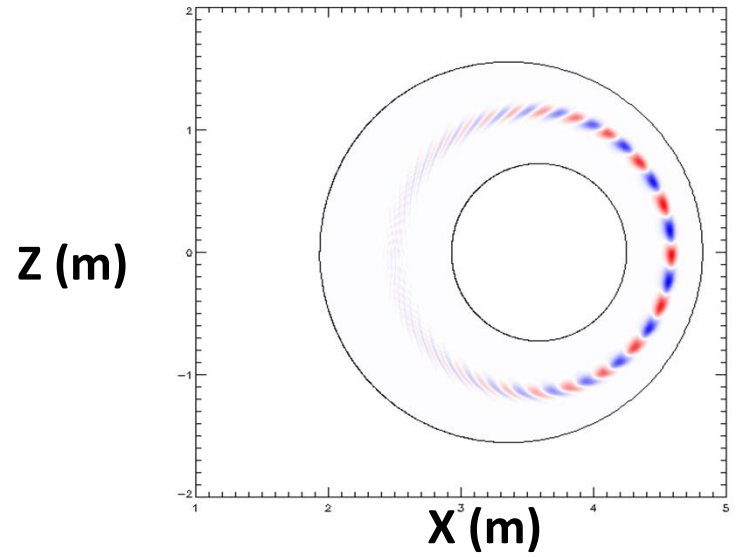
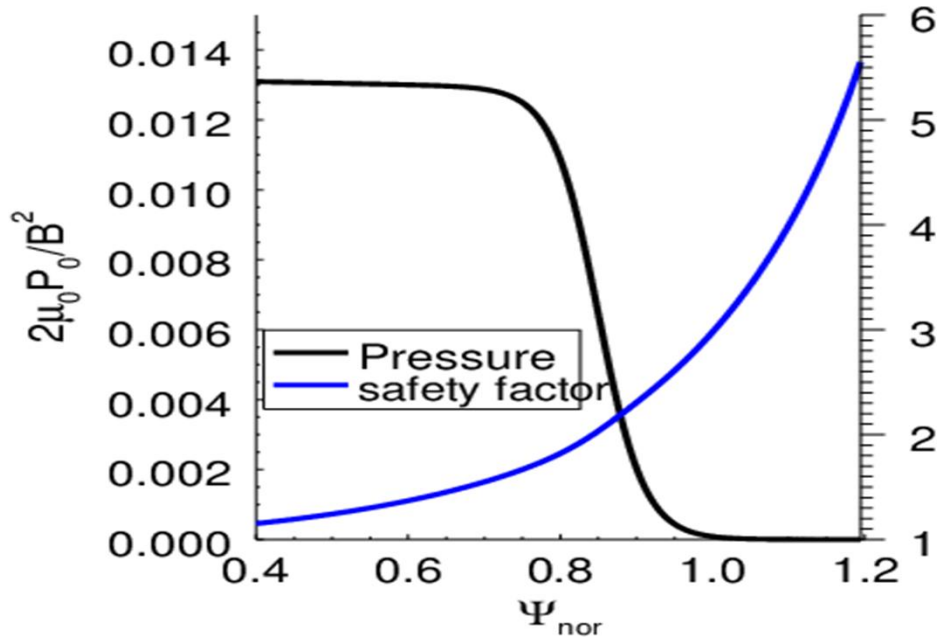
$$\frac{d\tilde{P}}{dt} + \mathbf{V}_E \cdot \nabla P_0 = 0$$

$$\frac{\partial A_{\parallel}}{\partial t} + \partial_{\parallel} \phi_T = \frac{\eta}{\mu_0} \nabla_{\perp}^2 A_{\parallel} + \frac{\eta_H}{\mu_0} \nabla_{\perp}^4 A_{\parallel}$$

$$\varpi = \frac{m_i n_0}{B} \left( \nabla_{\perp}^2 \phi + \frac{1}{en_0} \nabla_{\perp}^2 \tilde{P}_i \right)$$

hyper resistivity

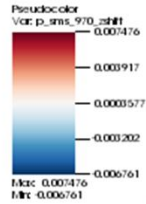
$$d/dt = \partial/\partial t + \mathbf{V}_{ET} \cdot \nabla, \mathbf{V}_{ET} = \frac{1}{R} \mathbf{b}_0 \times \nabla \phi_T, \phi_T = \phi_0 + \phi, \nabla_{\parallel} f = B \partial_{\parallel} \frac{f}{R}, \partial_{\parallel} = \partial_{\parallel}^0 + \delta \mathbf{b} \cdot \nabla, \delta \mathbf{b} = \frac{1}{B} \nabla A_{\parallel} \times \mathbf{b}_0, J_{\parallel} = J_{\parallel 0} + \tilde{J}_{\parallel}, \tilde{J}_{\parallel} = -\nabla_{\perp}^2 A_{\parallel} / \mu_0$$



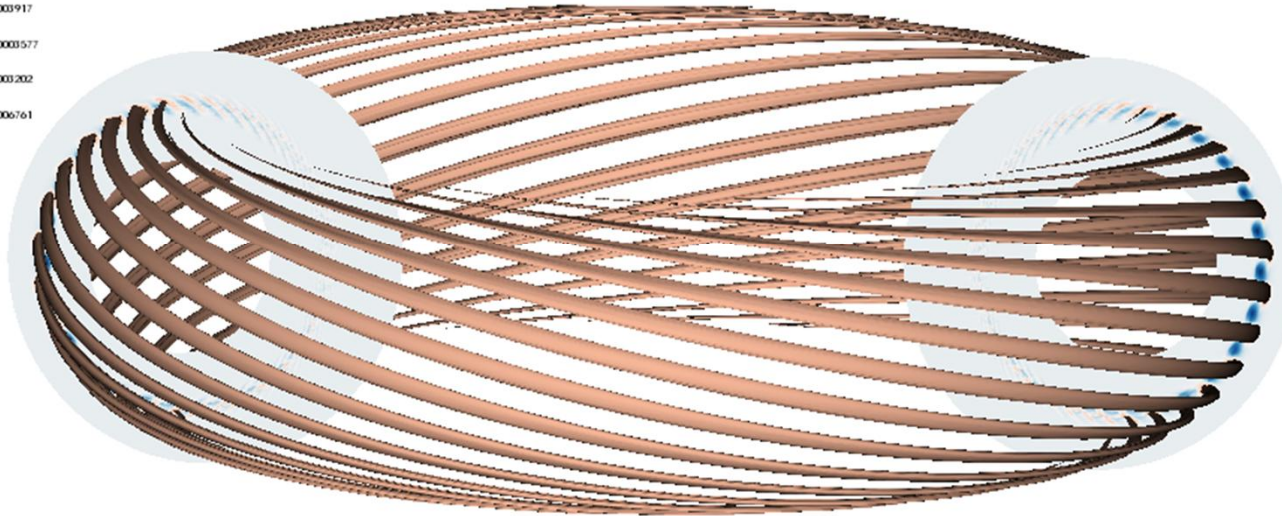
Comparison:

Single vs Multi-Mode Dynamics

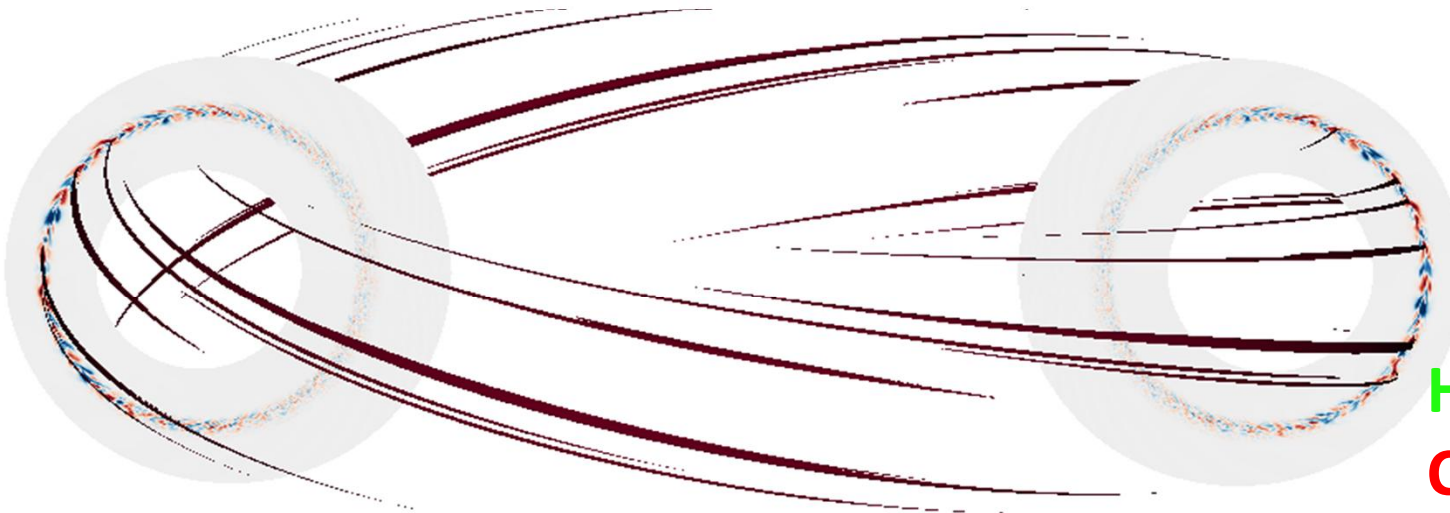
# 3D structure of pressure perturbation: filaments– helical coherent perturbation with outward radial motion



SMS → Filaments



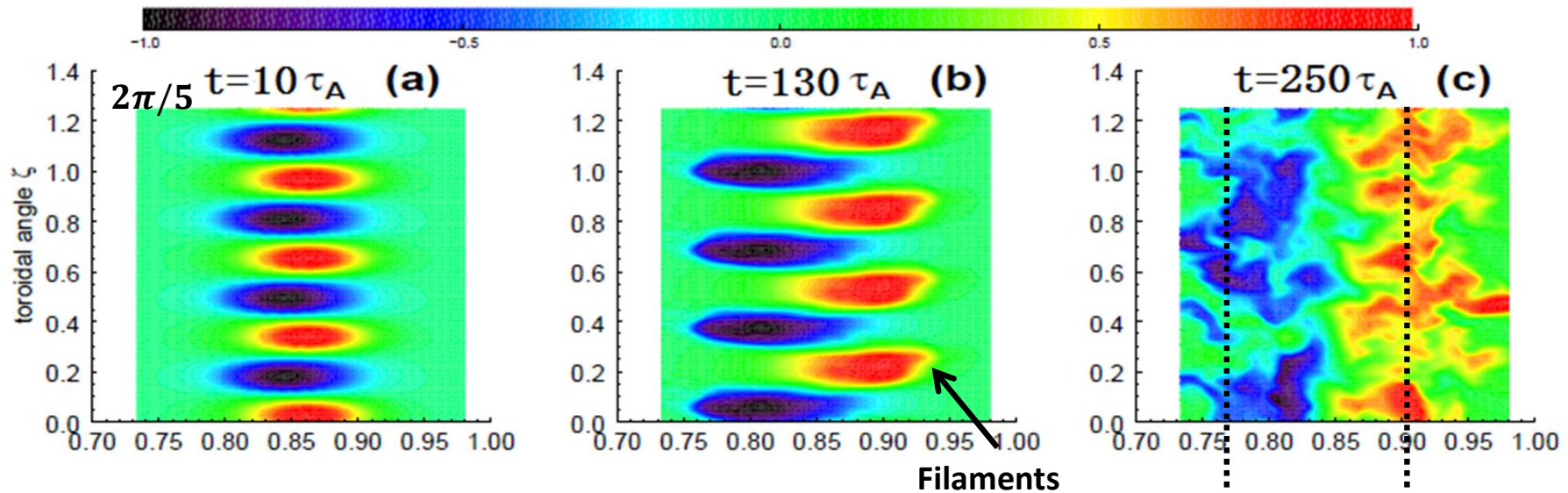
MMS → turbulence



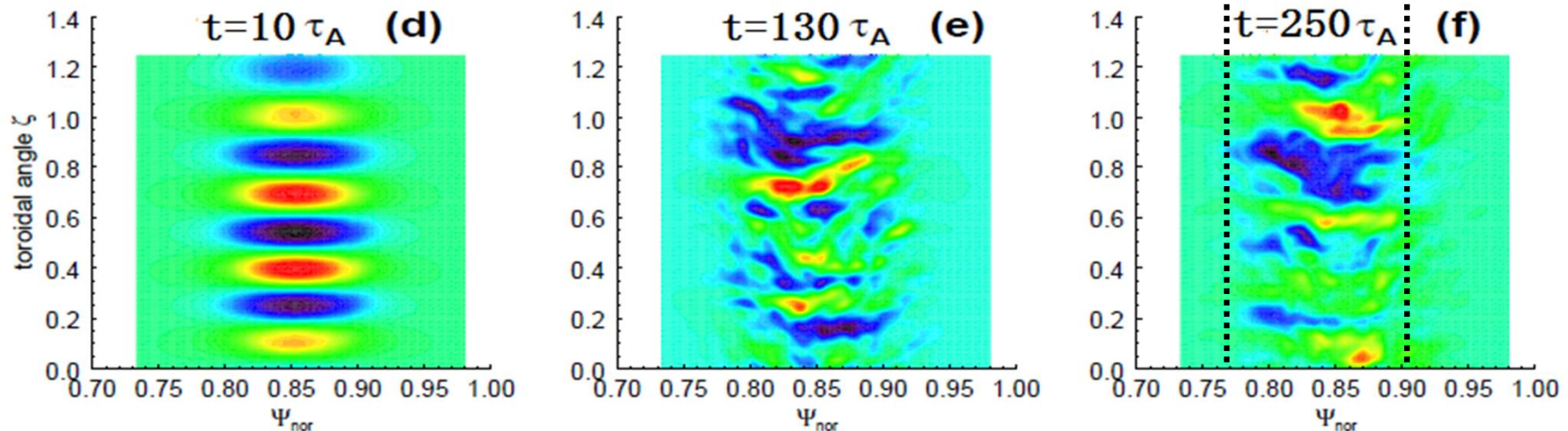
Helical  
Coherent  
radial motion

# Contrastive perturbation evolution (1/5 of the torus)

Single Mode



Multiple Mode



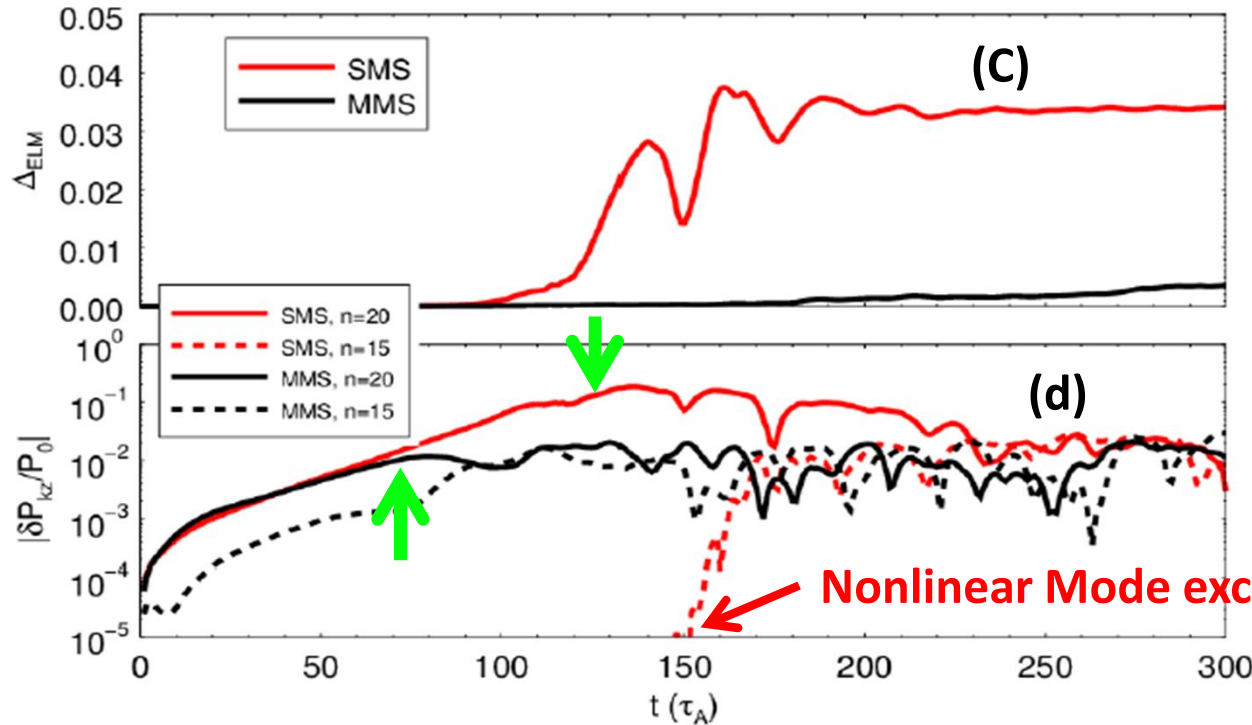
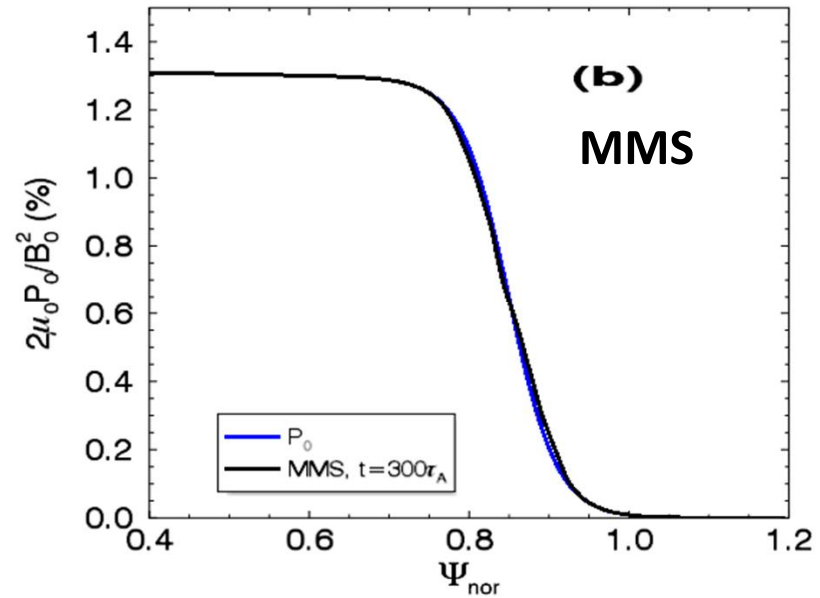
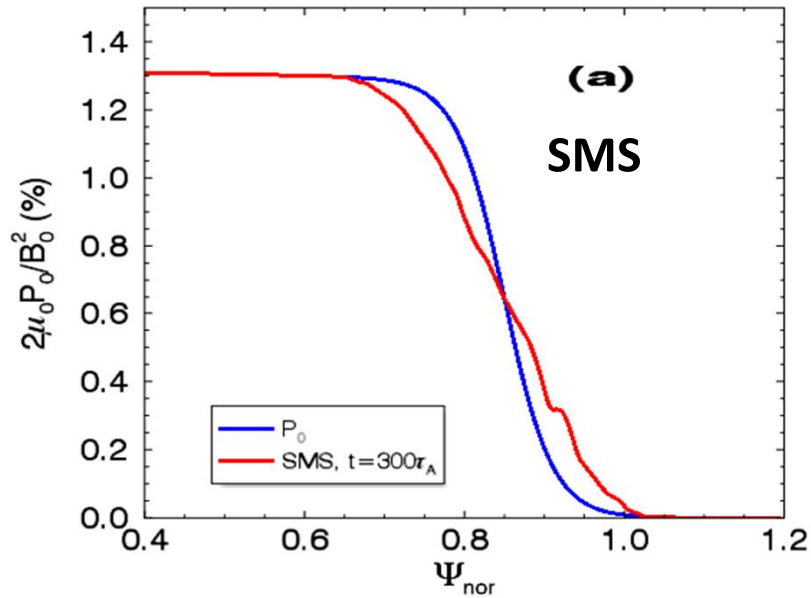
Linear phase

Early nonlinear phase

Late nonlinear phase

- Single mode: Filamentary structure is generated by linear instability;
- Multiple modes: Linear mode structure is disrupted by nonlinear mode interaction and no filamentary structure appears

# Single mode: ELM crash || Multiple modes: P-B turbulence



- ELM size larger for SMS

$$\Delta_{ELM} = \frac{\Delta W_{ped}}{W_{ped}} = \frac{\int dx^3 (P_0 - \langle P \rangle_{\zeta})}{\int dx^3 P_0}$$

- SMS has longer duration linear phase than MMS


# Relative **Phase** (Cross Phase) **Dynamics** and Peeling-Ballooning Amplification

# Peeling-Ballooning Perturbation Amplification is set by Coherence of Cross-Phase

i.e. schematic P.B. energy equation:

$$\frac{\partial}{\partial t} E_k = \langle \tilde{\phi} 2\hat{b}_0 \times \vec{k} \cdot \nabla \tilde{P} \rangle_{\vec{k}} \longleftarrow \sim \langle \tilde{v}_r \tilde{P} \rangle \rightarrow \begin{array}{l} \text{energy release from } \nabla \langle P \rangle \\ \rightarrow \text{quadratic} \end{array}$$

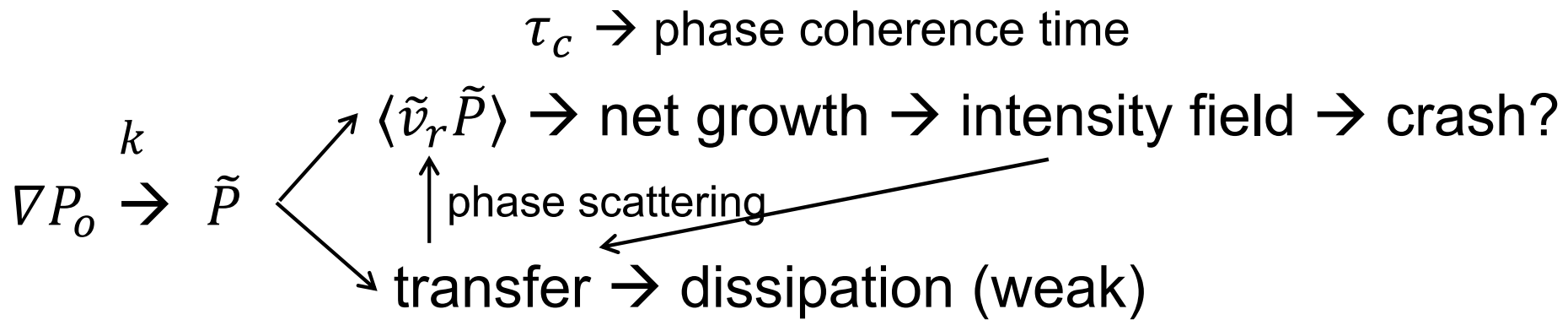
$$+ \sum_{\vec{k}', \vec{k}''} \tau_{c\vec{k}} C(\vec{k}', \vec{k}'') E_{\vec{k}'} E_{\vec{k}''} - \sum_{\vec{k}'} \tau_{c\vec{k}+\vec{k}'} C(\vec{k}', \vec{k}) E_{\vec{k}'} E_{\vec{k}} - \text{dissipation}$$


  
 nonlinear mode-mode coupling  $\rightarrow$  quartic

NL effects

- energy couplings to transfer energy (weak)
- response scattering to de-correlate  $\tilde{\phi}$ ,  $\tilde{P} \rightarrow$  regulate drive

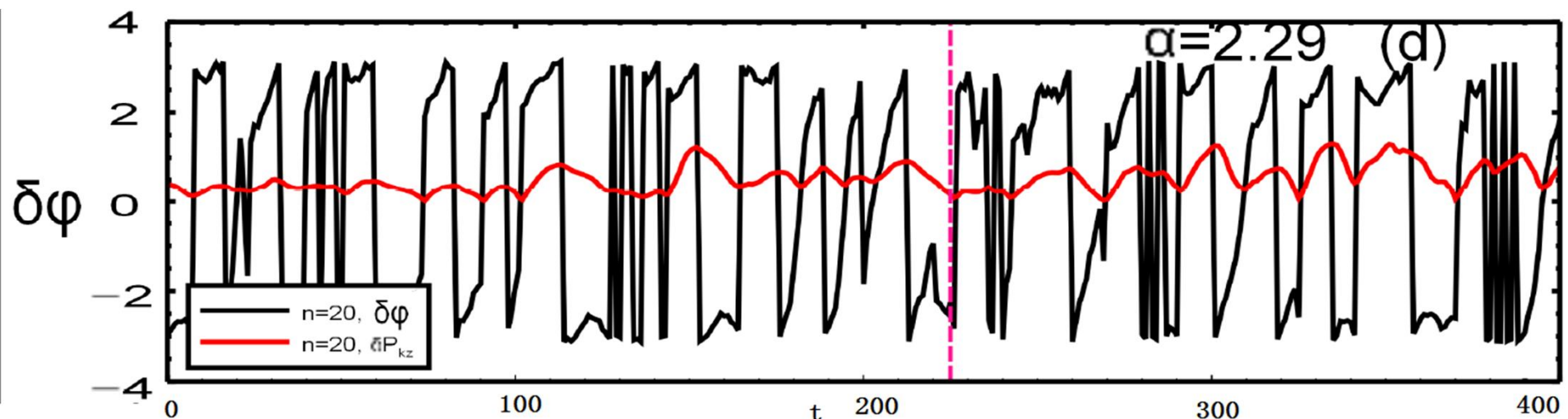
# Growth Regulated by Phase Scattering



Critical element: relative phase

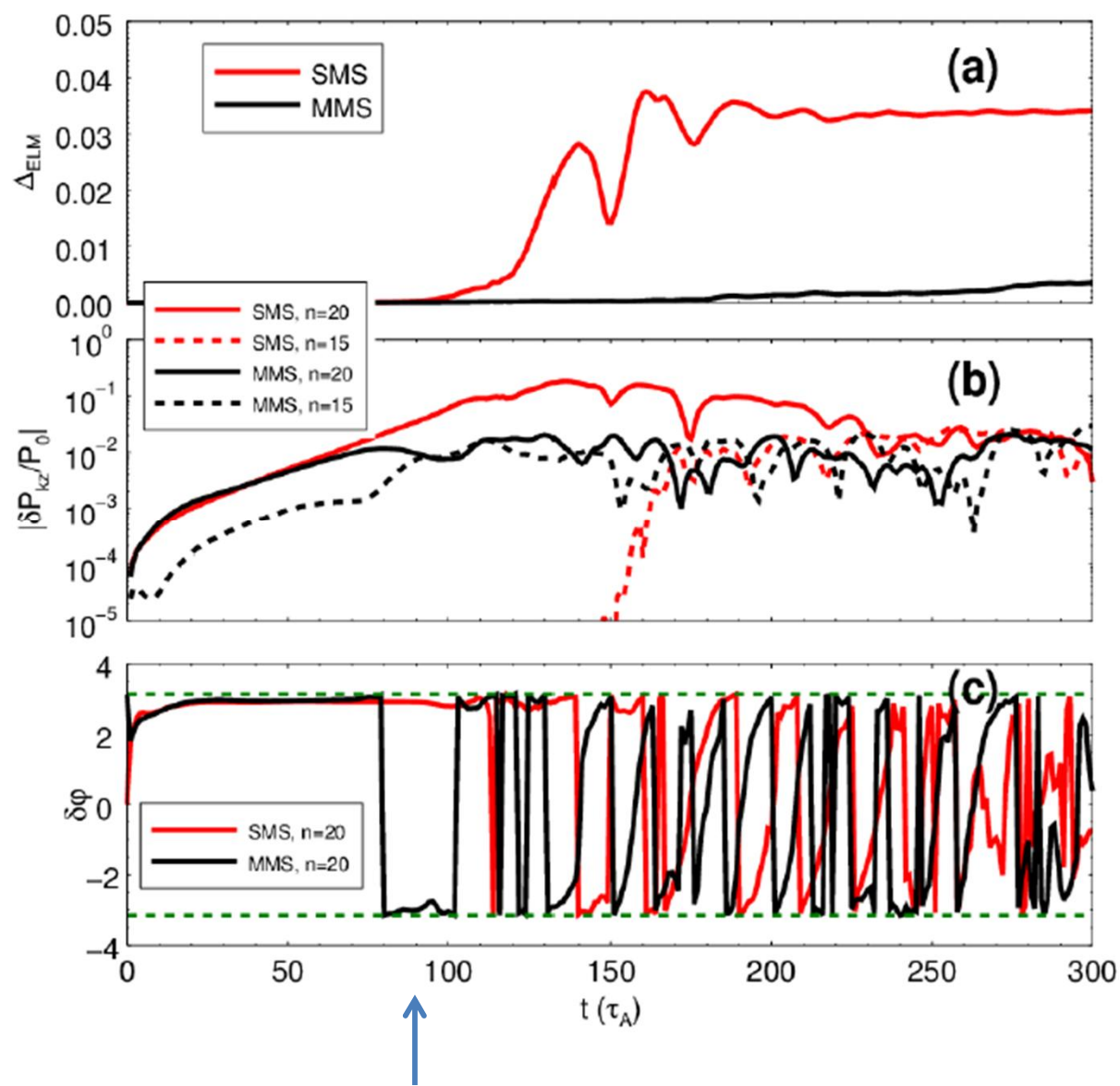
$$\delta\varphi = \text{arg} [\hat{p}_n / \hat{\phi}_n]$$

Phase coherence time sets growth





# Cross Phase Exhibits Rapid Variation in Multi-Mode Case



- Single mode case  $\rightarrow$  coherent phase set by linear growth  $\rightarrow$  rapid growth to 'burst'
- Multi-mode case  $\rightarrow$  phase de-correlated by mode-mode scattering  $\rightarrow$  slow growth to turbulent state

# Key Quantity: Phase Correlation Time

- Ala' resonance broadening (Dupree '66):

$$\frac{\partial}{\partial t} \hat{P} + \tilde{v} \cdot \nabla \hat{P} + \langle v \rangle \cdot \nabla \hat{P} - D \nabla^2 \hat{P} = -\tilde{v}_r \frac{d}{dr} \langle P \rangle$$

Nonlinear  
scattering

Linear streaming  
(i.e. shear flow)

Ambient  
diffusion

$$\hat{P} = A e^{i\phi}$$

Relative phase  $\leftrightarrow$  cross-phase

Amplitude

$$\hat{v} = B$$

Velocity amplitude

$$\rightarrow \partial_t \tilde{\phi} + \tilde{v} \cdot \nabla \tilde{\phi} + \langle v(r) \rangle \cdot \nabla \tilde{\phi} - D \nabla^2 \tilde{\phi} - \frac{2D}{A} \nabla A \cdot \nabla \tilde{\phi} = 0$$

NL scattering      shearing

$$\partial_t A + \tilde{v} \cdot \nabla A + \langle v(r) \rangle \cdot \nabla A + D (\nabla \tilde{\phi})^2 A - D \nabla^2 A = -B \frac{d}{dr} \langle P \rangle$$

Damping by phase fluctuations

# Phase Correlation Time

- Stochastic advection:

$$\frac{1}{\tau_{ck}} = \vec{k} \cdot D_\phi \cdot \vec{k} + k^2 D$$

$$D_\phi = \sum_{k'} \tau_{ck'} |\tilde{v}'_{\perp k}|^2$$

- Stochastic advection + sheared flow:

$$\frac{1}{\tau_{ck}} \approx \left( k_\perp^2 (D_\phi + D) \langle v_\perp \rangle'^2 \right)^{1/3} \quad \rightarrow \text{Coupling of radial scattering and Shearing shortens phase correlation}$$

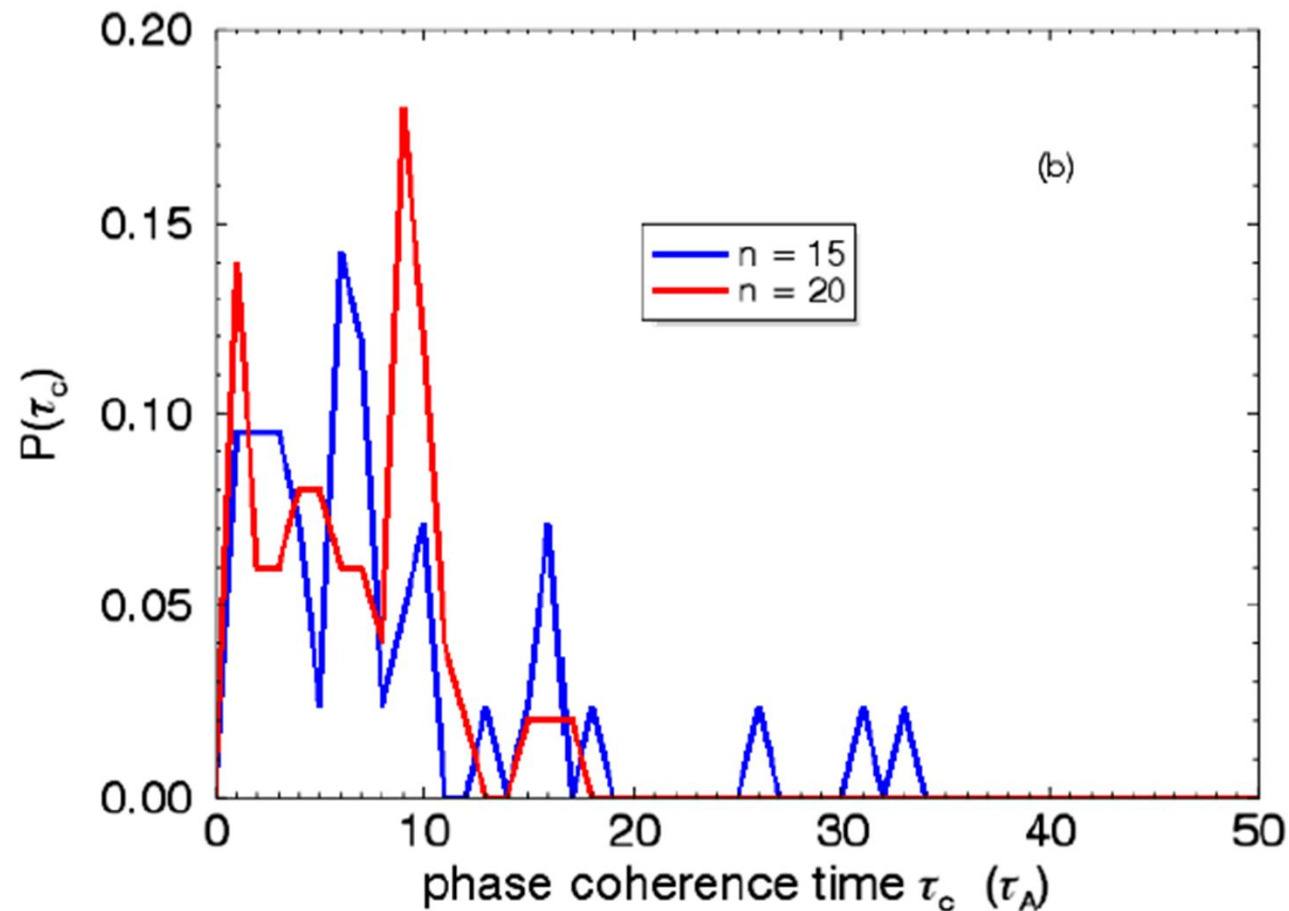
- Parallel conduction + diffusion:

$$\frac{1}{\tau_{ck}} \approx \left[ \frac{\hat{s}^2 k_\perp^2}{(Rq)^2} \chi_\parallel (D_\phi + D) \right]^{1/2} \quad \rightarrow \text{Coupling of radial diffusion and conduction shortens phase correlation}$$

# What is actually known about fluctuations in relative phase?

- For case of P.-B. turbulence, a broad PDF of phase correlation times is observed

pdf  
of  $\tau_c$



Implications for: i) Bursts vs Turbulence  
ii) Threshold

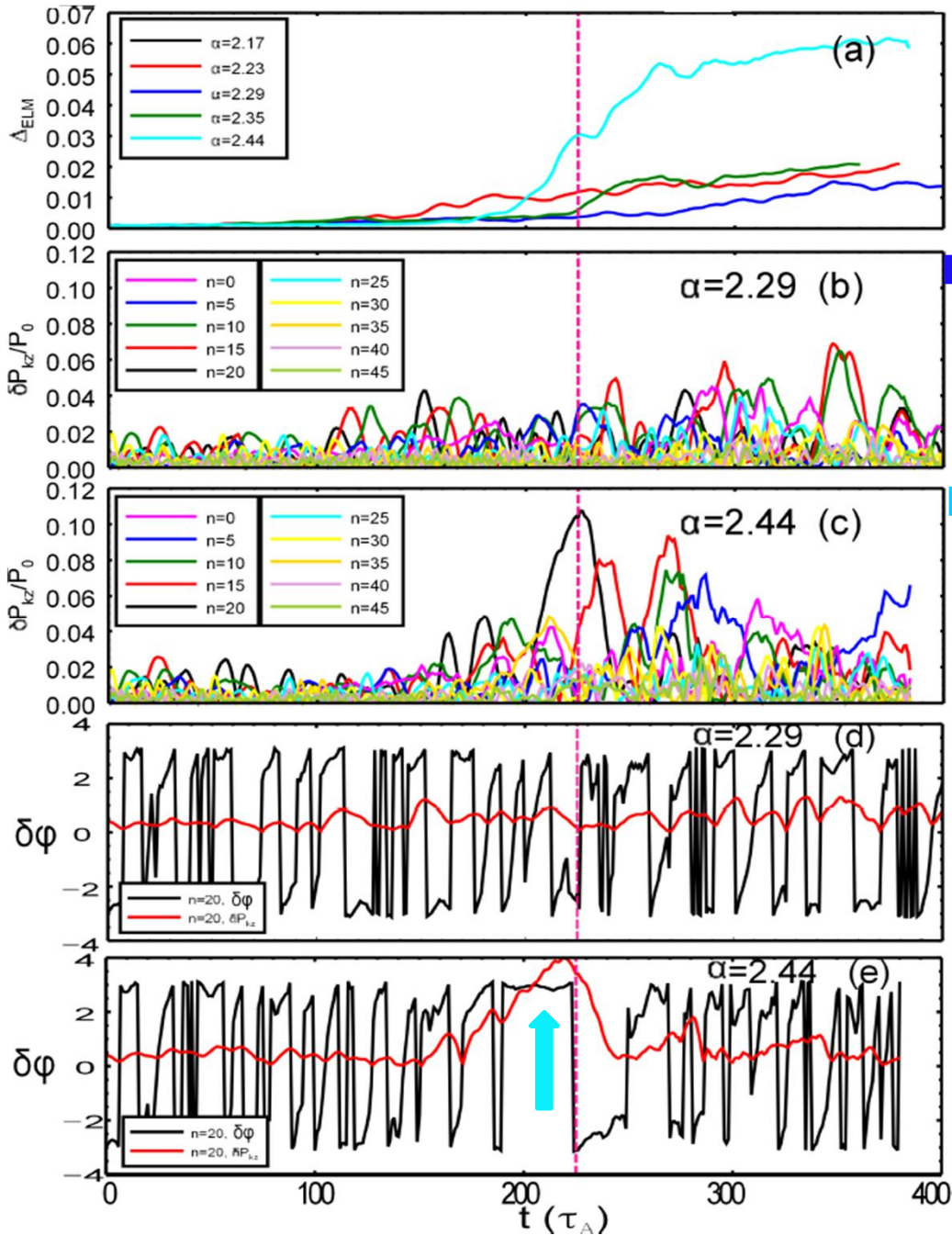
# Bursts, Thresholds

- P.-B. turbulence can scatter relative phase and so reduce/limit growth of P.-B. mode to large amplitude
- Relevant comparison is:

$$\gamma_k^L \text{ (linear growth)} \quad \text{vs} \quad \frac{1}{\tau_{ck}} \text{ (phase de-correlation rate)}$$

- Key point: Phase scattering for mode  $\vec{k}$  set by ‘background modes  $\vec{k}'$ ’ i.e. other P.-B.’s or micro-turbulence  
→ is the background strong enough??

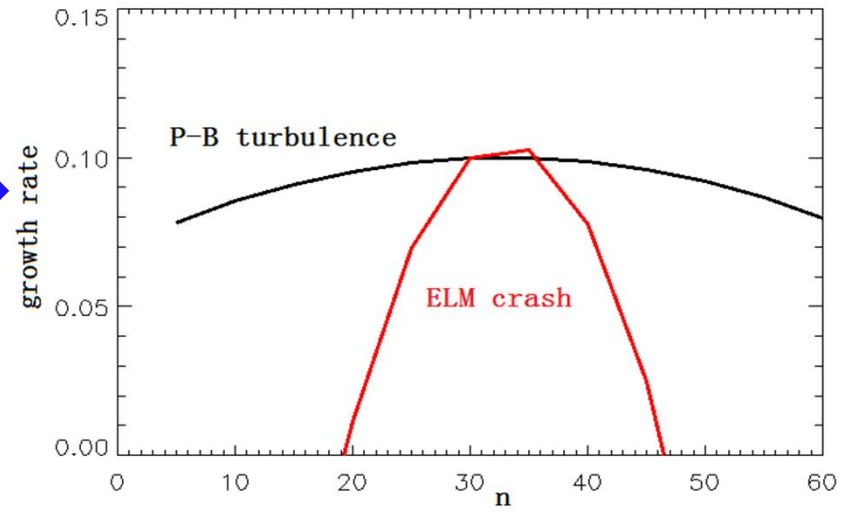
# The shape of growth rate spectrum determines burst or turbulence



**P-B turbulence**  
 $\gamma(n)\tau_c(n) < \ln 10$

**Isolated ELM crash**  

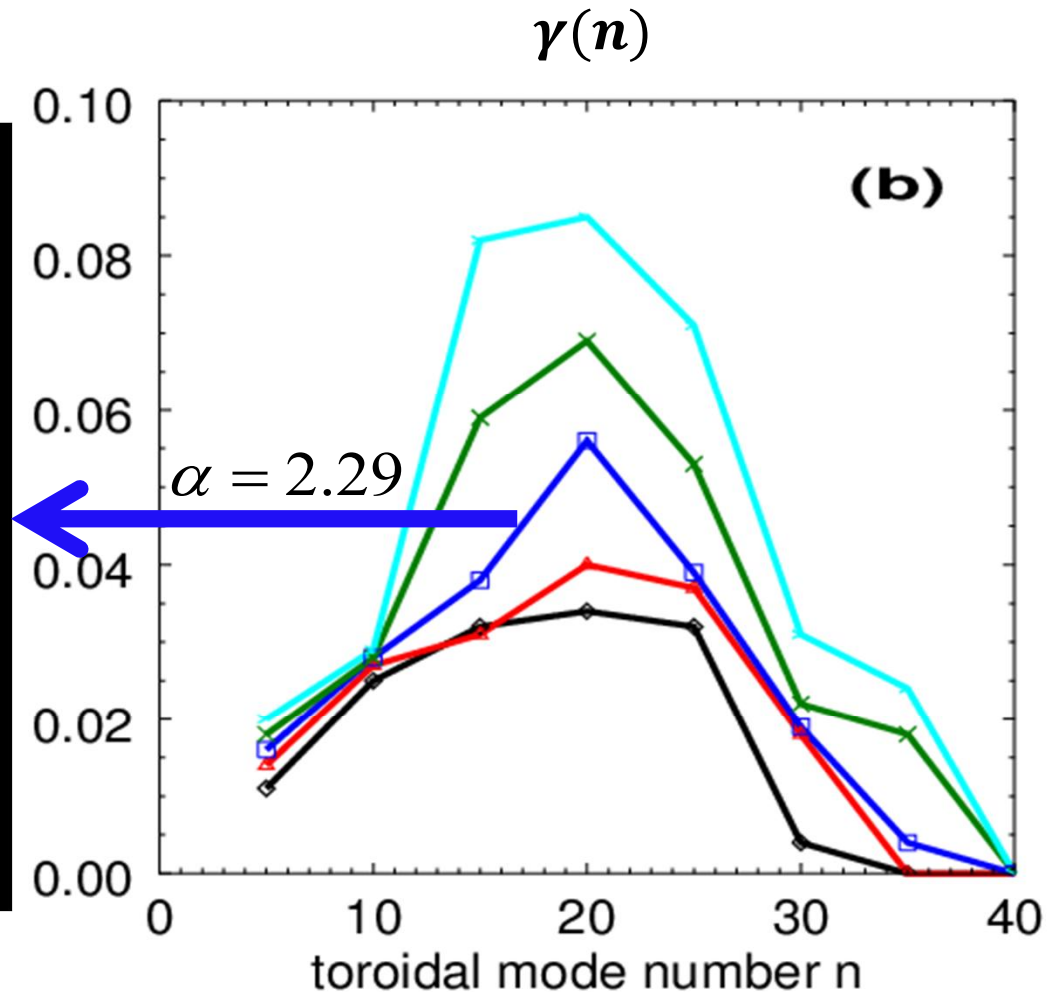
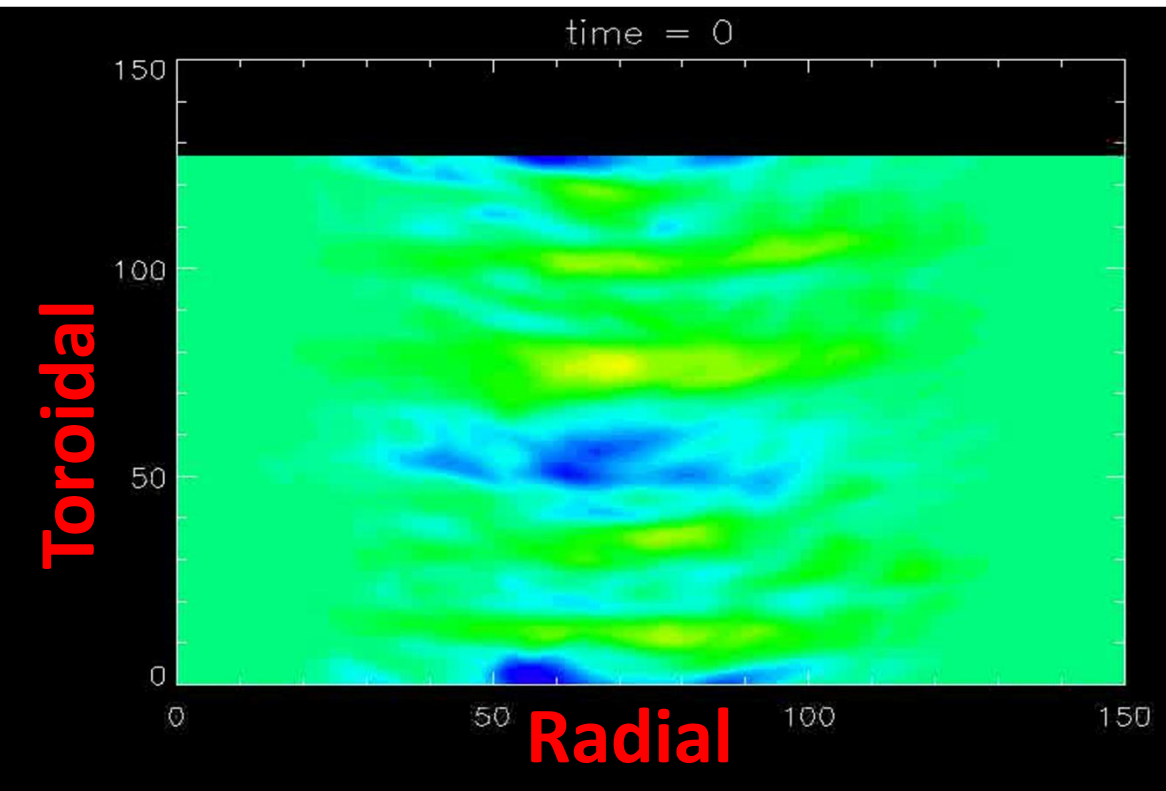
$$\begin{cases} \gamma(n)\tau_c(n) > \ln 10, n = n_{dom} \\ \gamma(n)\tau_c(n) < \ln 10, n \neq n_{dom} \end{cases}$$



So When Does it Crash?



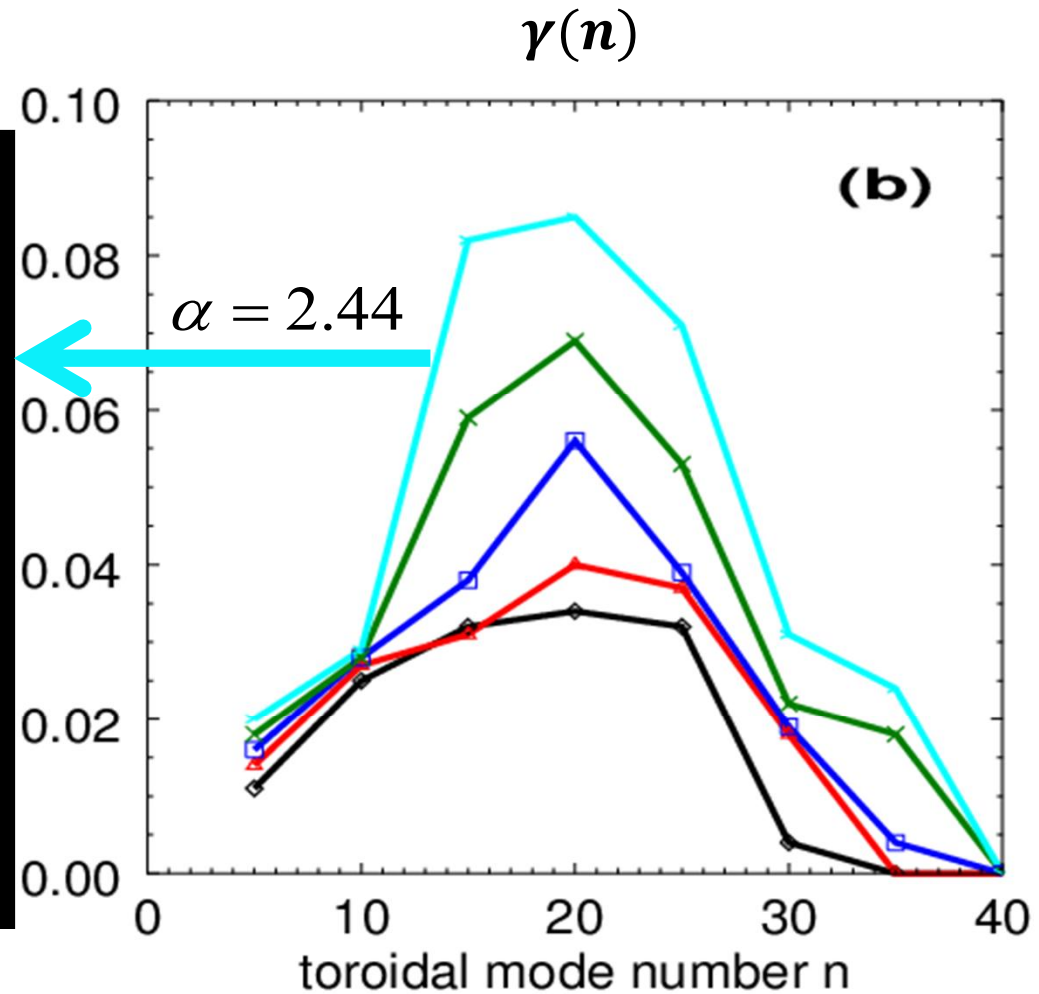
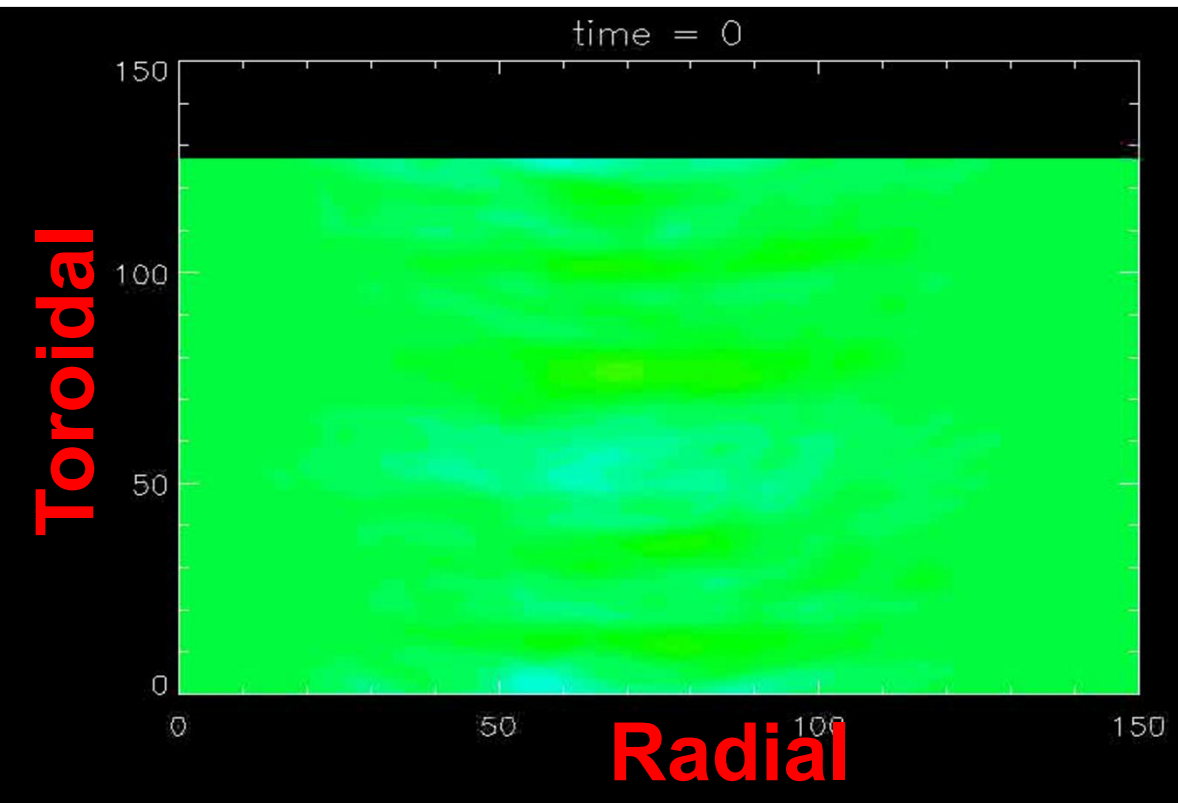
# Modest $\gamma(n)$ Peaking $\rightarrow$ P.-B. turbulence



- Evolution of P-B turbulence
  - No filaments
  - Weak radial extent

$$\alpha = -2\mu_0 R P_0' q^2 / B^2$$

# Stronger Peaking $\gamma(n) \rightarrow$ ELM Crash

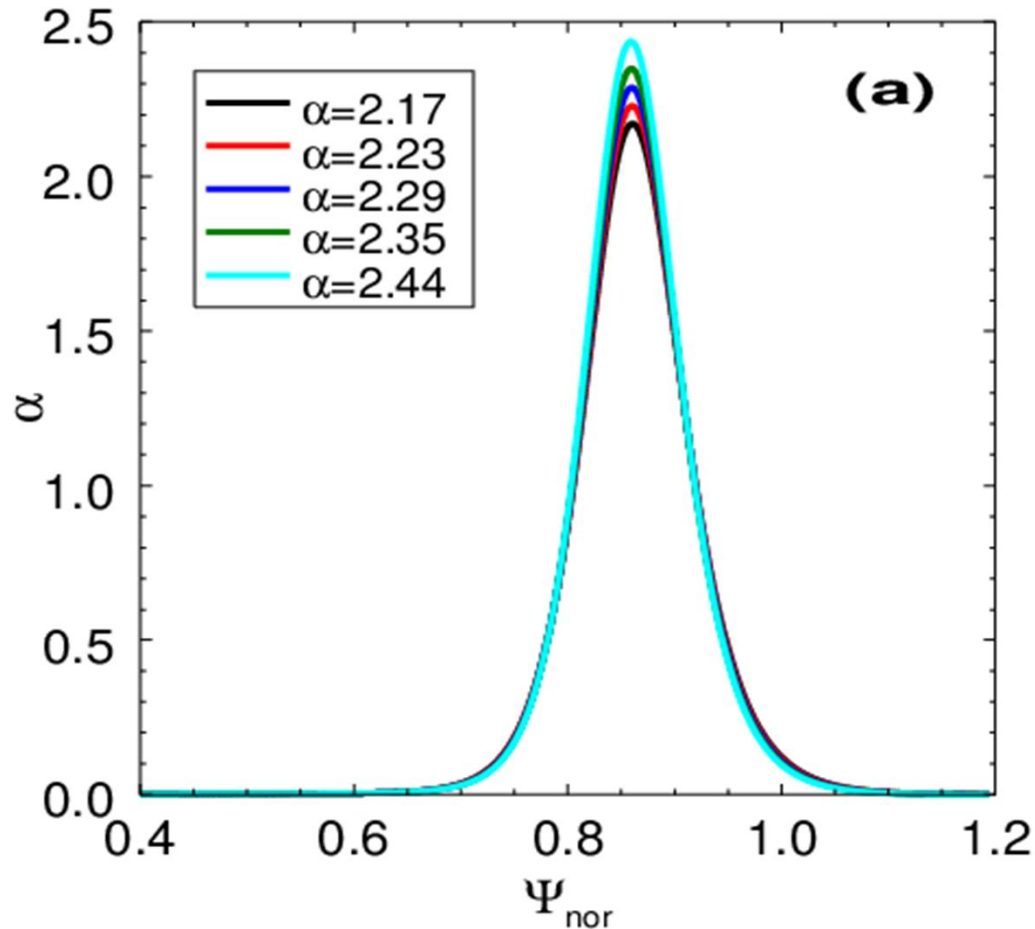


- ELM crash is triggered
- Wide radial extension

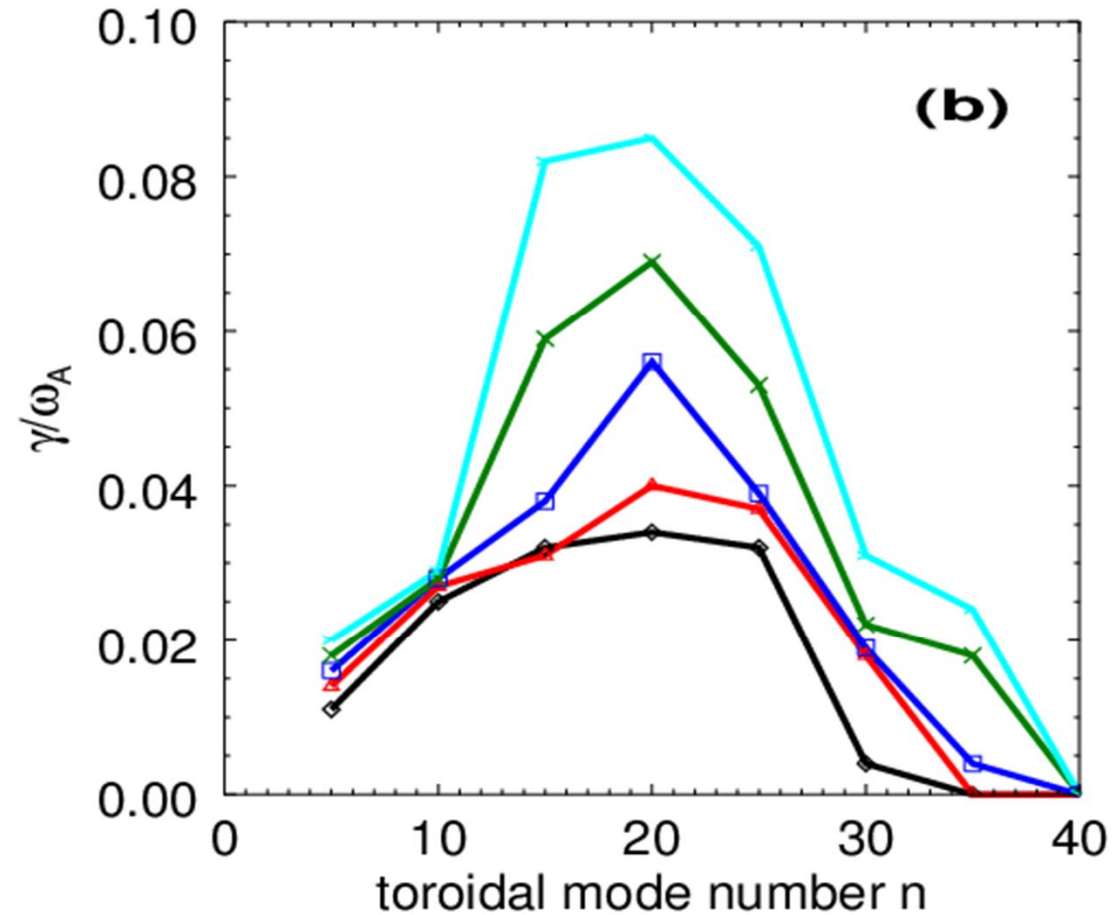
$$\alpha = -2\mu_0 R P_0' q^2 / B^2$$

# $\gamma(n)$ Peaking VERY Sensitive to Pressure Gradient

Normalized pressure gradient



$\gamma(n)$



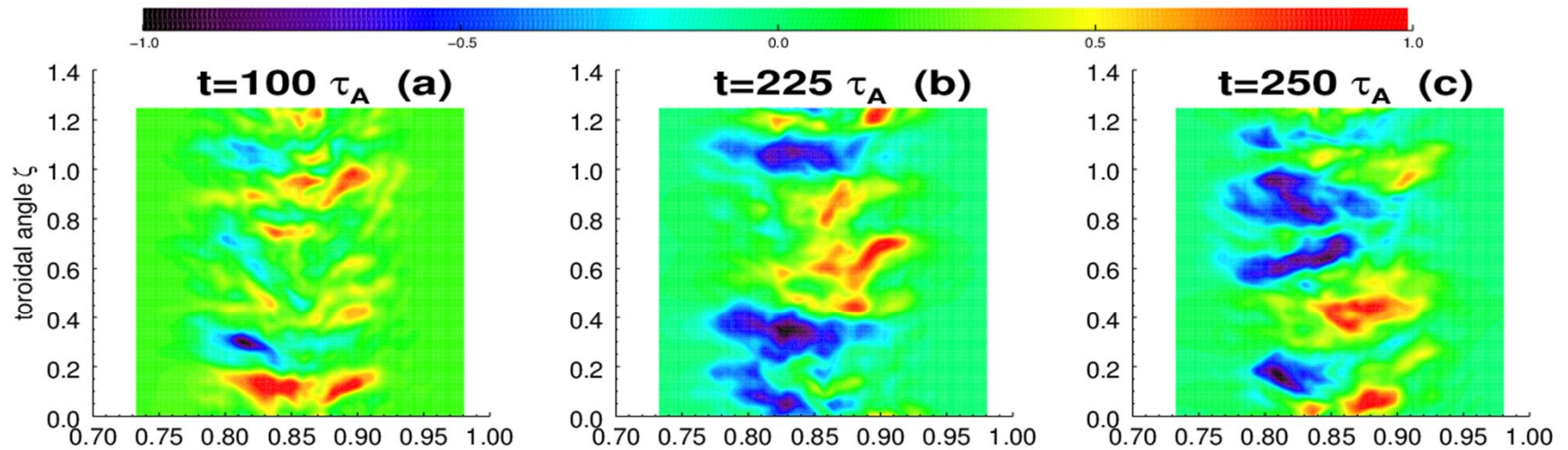
$$\alpha = -2\mu_0 R P_0' q^2 / B^2$$

◆ Higher pressure gradient

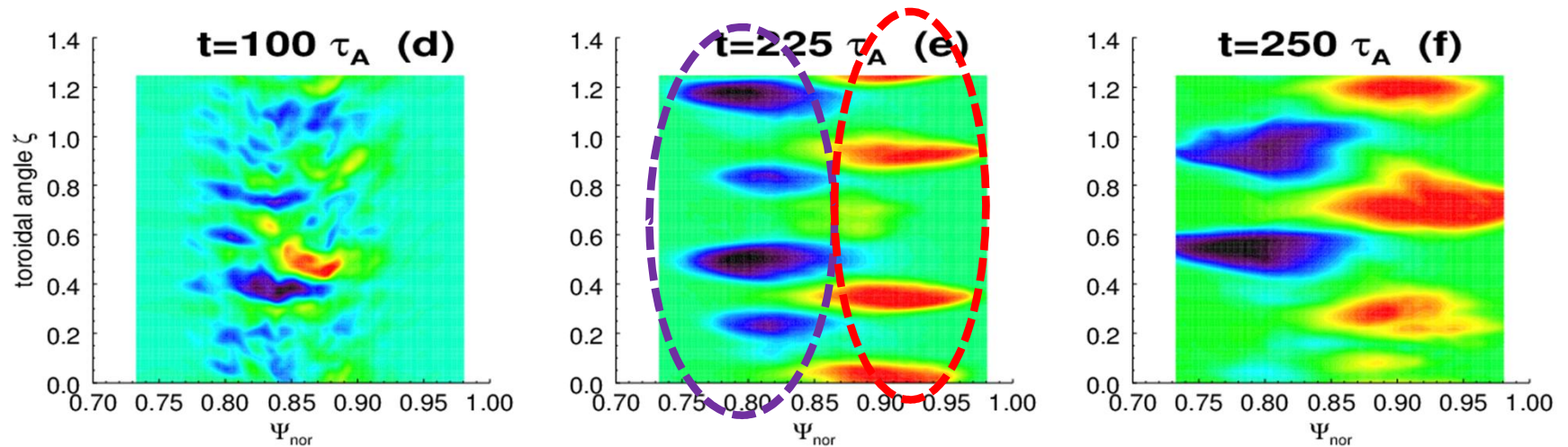
- ✓ Larger growth rate;
- ✓ Peaking of growth rate spectrum;

# Filamentary structure may not correspond to that of the most unstable mode due to nonlinear interaction

$\alpha = 2.29$   
P-B turbulence



$\alpha = 2.44$   
ELM crash



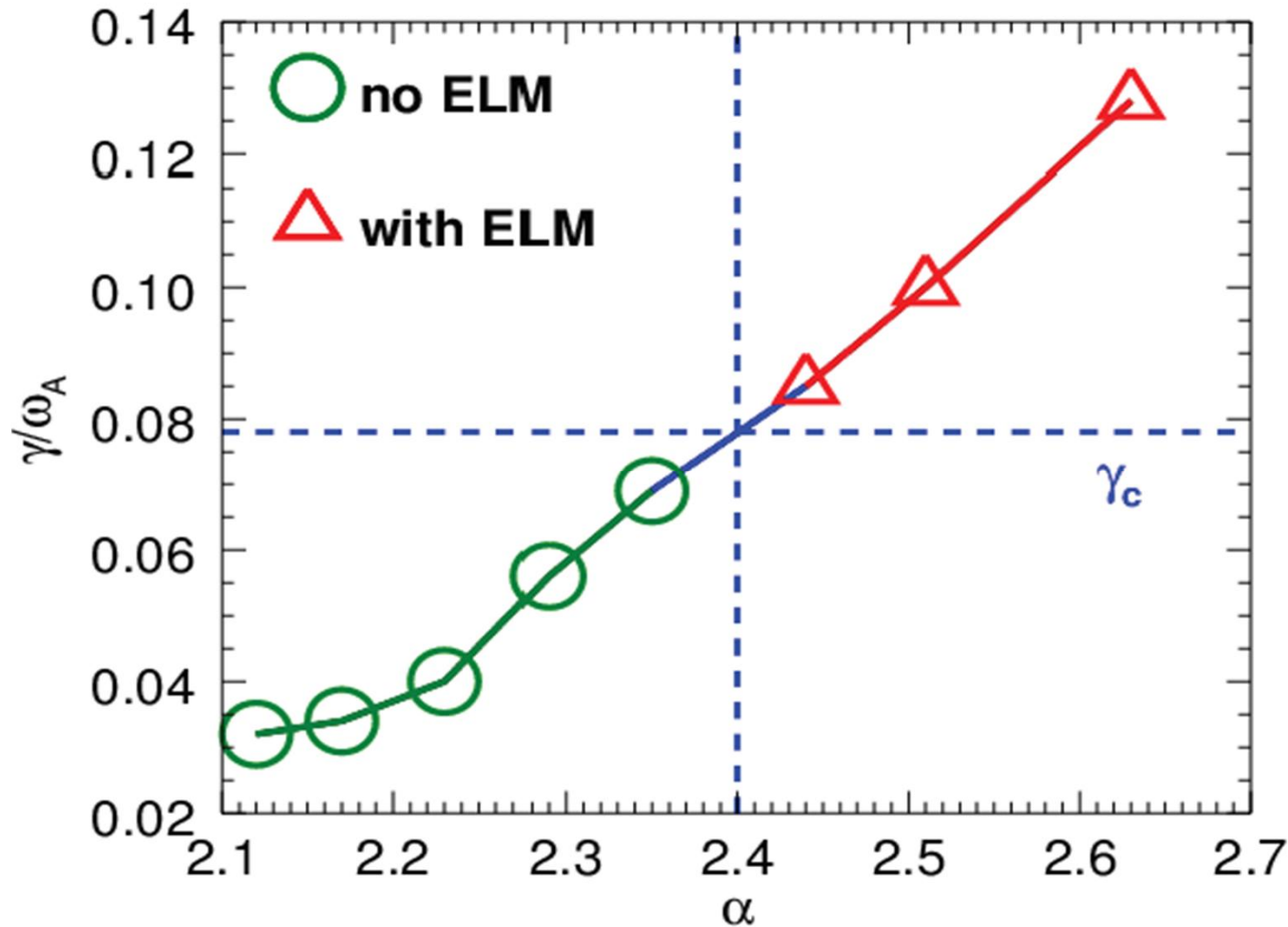
❑ **Triggering ELM and the generation of filamentary structure are different processes!**

- ✓ ELM is triggered by the most unstable mode;
- ✓ Filamentary structure depends on both linear instability and nonlinear mode interaction.

What is the Threshold for a Crash?

Linear criterion for the onset of ELMs  $\gamma > 0$  is replaced by the nonlinear criterion

$$\gamma > \gamma_c \sim 1/\tau_c$$



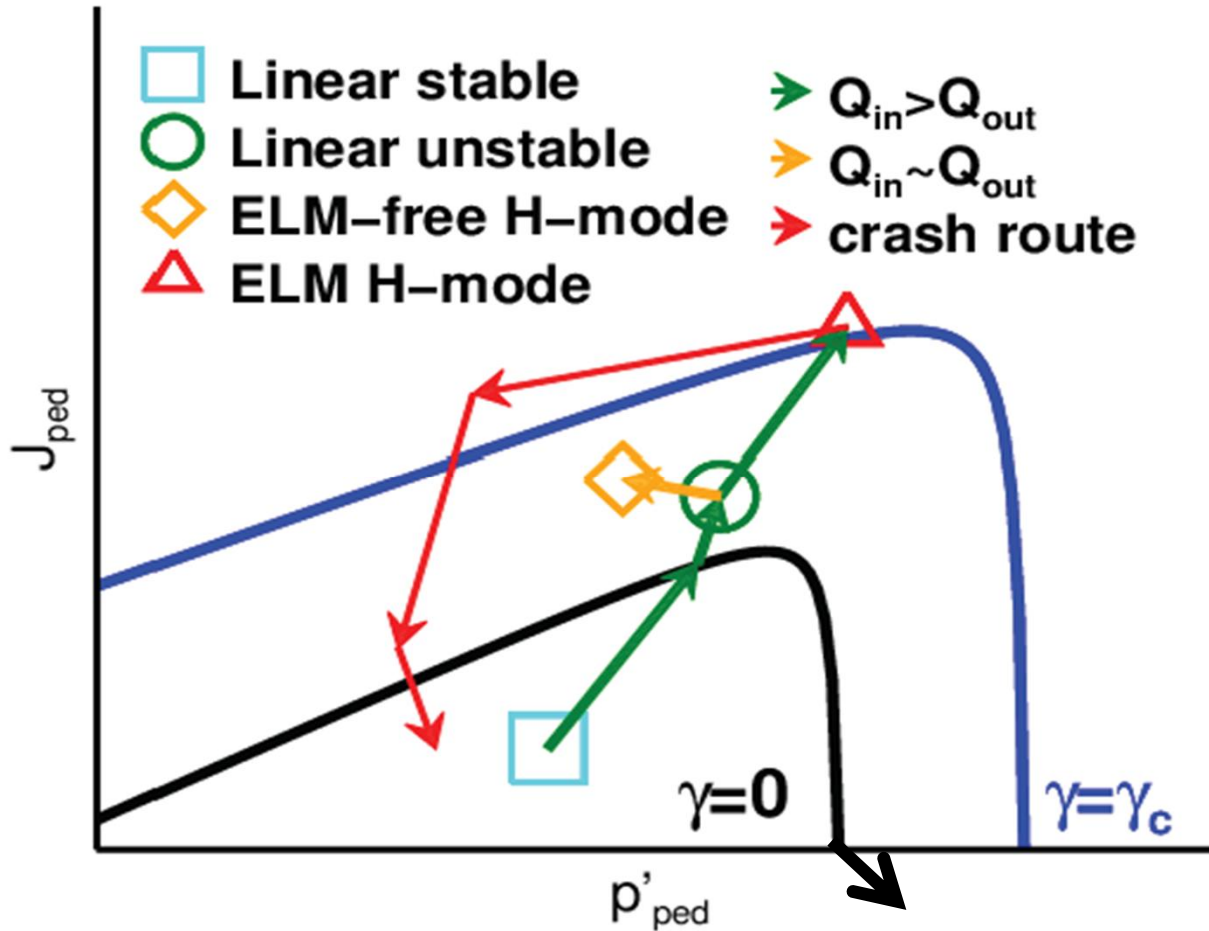
- Criterion for the onset of ELMs  
 $\gamma\tau_c > \ln 10 \Rightarrow \gamma > \frac{\ln 10}{\tau_c} \equiv \gamma_c$

- Linear limit

$$\lim_{\tau_c \rightarrow \infty} \Rightarrow \gamma > 0$$

- $\gamma_c$  is the critical growth rate which is determined by nonlinear interaction in the background turbulence
- N.B.  $1/\tau_c$  - and thus  $\gamma_{crit}$  - are functionals of  $\gamma_L(n)$  peakedness

# Nonlinear Peeling-ballooning model for ELM:



- $\gamma < 0$ :  
Linear stable region
- $0 < \gamma < \gamma_c$ : Turbulent region  
Possible ELM-free regime →  
Special state: EHO, QCM (?!)
- $\gamma > \gamma_c$ :  
ELMy region

✓ Different regimes depend on **both linear instability and the turbulence** in the pedestal.

Including all relevant linear physics  
(not only ideal P-B with  $\omega_*$ )  
Resistivity / Electron inertia / ...

➔ **Turbulence can maintain ELM-free states**

# Partial Summary

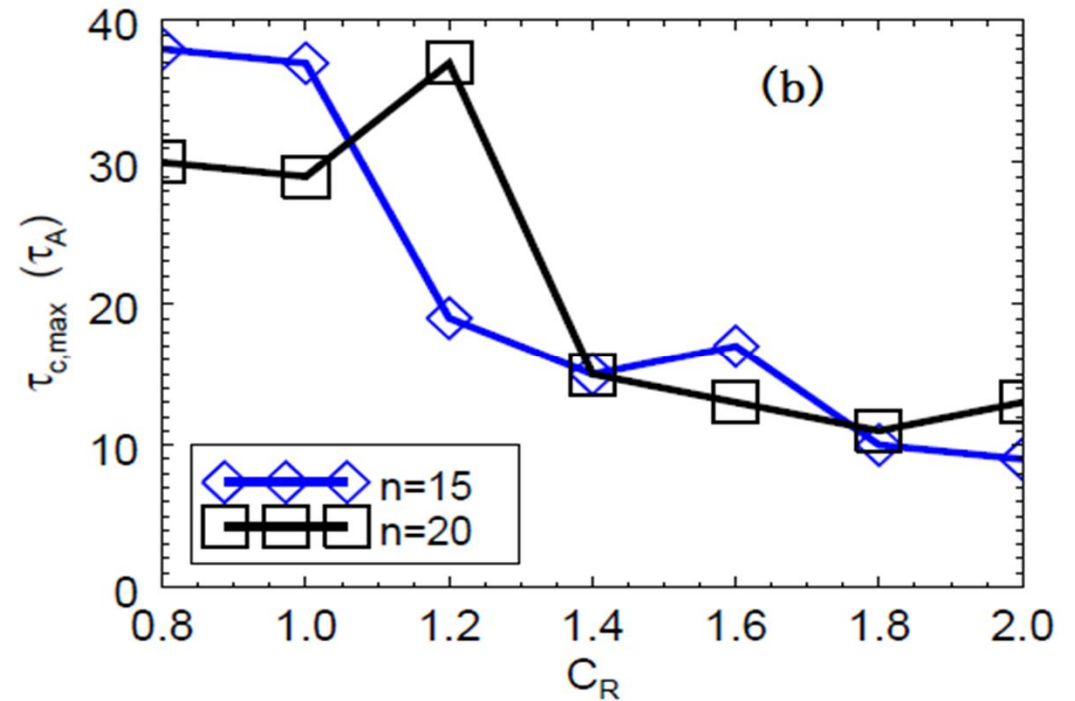
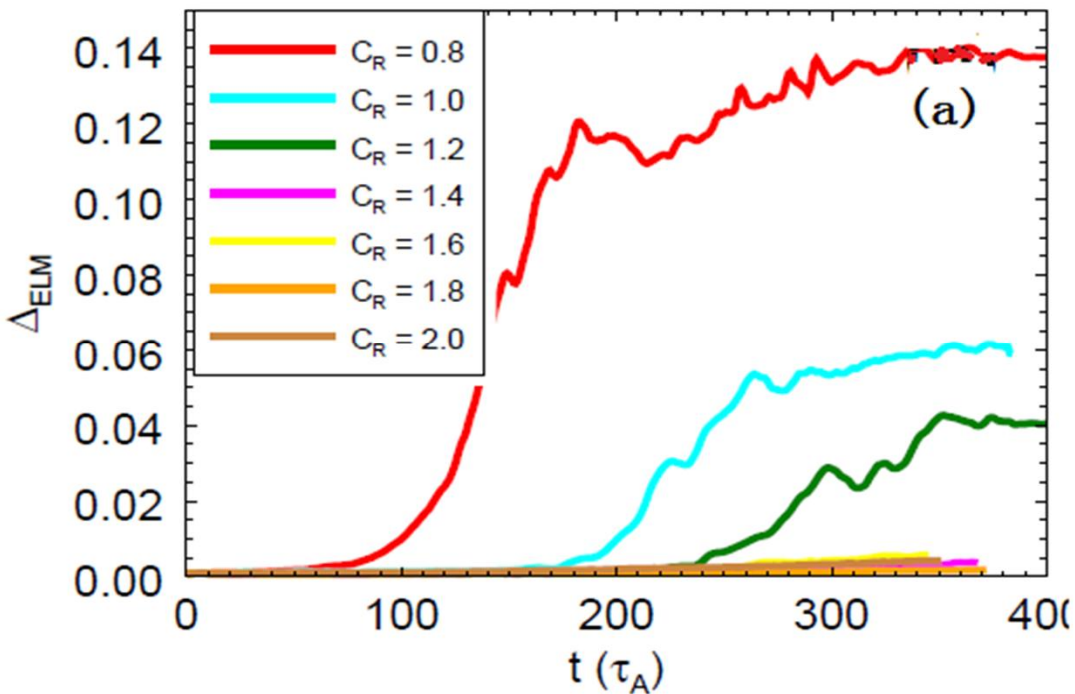
- Multi-mode P.-B. turbulence or  $\sim$  coherent filament formation can occur in pedestal
- Phase coherence time is key factor in determining final state and net P.-B. growth
- Phase coherence set by interplay of nonlinear scattering with ‘differential streaming’ in  $\hat{P}$  response
- Key competition is  $\gamma_L$  vs  $1 / \tau_c \rightarrow$  defines effective threshold
- Peekedness of  $\gamma(n)$  determines burst vs turbulence



How can these ideas be exploited  
for ELM mitigation and control?

# ELMs can be controlled by reducing phase coherence time

$$\frac{\partial \varpi}{\partial t} + C_R \frac{\mathbf{b} \times \nabla \phi}{B} \cdot \nabla \varpi = RHS \quad \text{i.e. scan } C_R \text{ for fixed profiles}$$



- ELMs are determined by the product  $\gamma(n)\tau_c(n)$ ;
- Reducing the phase coherence time can limit the growth of instability;
- **Different turbulence states lead to different phase coherence times and, thus different ELM outcomes**

# Keys to $\tau_c$

- Scattering field
- ‘differential rotation’ in  $\hat{P}$  response to  $\hat{v}_r$ 
  - enhanced phase de-correlation

## Knobs:

- ExB shear
- Shaping
- Ambient diffusion
- Collisionality

## Mitigation States:

- QH mode, EHO
- RMP
- SMBI
- ...

# Scenarios

- QH-mode

- enhanced ExB shear  $\rightarrow \frac{1}{\tau_c} \rightarrow (k_{\perp}^2 D \langle V_E \rangle^2)^{1/3}$
- Triangularity strengthens shear via flux compression
- Enhanced de-correlation restricts growth time

Also:

- Is EHO peeling/kink + reduced  $\tau_c$ ?
- $\langle V_E \rangle'$  works via  $\gamma_L$  and  $\tau_c$

N.B. See Bin Gui, Xu; this meeting for more on shearing effects

# Scenarios

- RMP

- $\frac{1}{\tau_c} = \left( \frac{k_{\perp}^2 \hat{s}^2}{(Rq)^2} \chi_{\parallel} D \right)^{1/2} \quad D = D_{\phi} + D_{amb}$

- RMP  $\rightarrow D_{amb} \uparrow \rightarrow$  enhanced de-correlation

or

- Enhanced flow damping  $\rightarrow$  enhanced turbulence  $\rightarrow$  increased  $D_{\phi}$

- SMBI

- enhanced  $D_{\phi} \rightarrow$  reduced  $\tau_c$  ?

and/or

- Disruption of pedestal avalanches?

## II) Reconnection and Hyper-resistivity

# Some Basics

- P.-B.'s are ideal modes  $\leftrightarrow$  frozen-in law... !?
- ELM phenomena requires **irreversibility** for:
  - field-fluid decoupling, reconnection
  - Transport, cross field
  - Magnetic stochastization
- What is mechanism of fast reconnection for ELM? Resistivity unlikely...
- $S \geq 10^8$  in pedestal  $\rightarrow$  **hyper-resistivity** becomes natural candidate

- Hyper-resistivity!? – Electron momentum transport

i.e.  $E_{\parallel} = \eta J_{\parallel} + \nabla_{\perp} \mu_H \nabla_{\perp} \hat{J}_{\parallel}$  ←

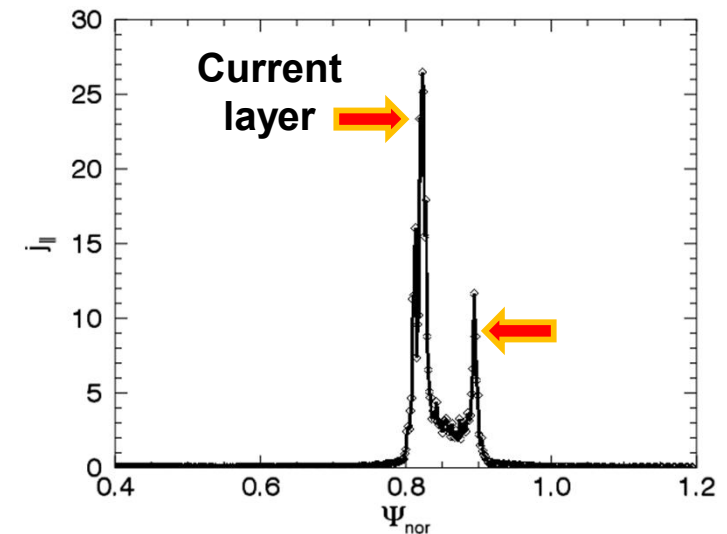
⊥ transport of parallel current - ambient micro-turbulence  
 - P.-B. turbulence

- Xu et al 2010 → Hyper-resistivity  $\sim \chi_e$  needed to dissipate current sheets, so as allow ELM crash
- Hyper-resistivity generally can trigger fast reconnection

i.e. Sweet-Parker: - resistive:  $\frac{V}{V_A} \sim \frac{1}{\sqrt{R_m}}$

- hyper-resistive:  $\frac{V}{V_A} \sim \frac{1}{(R_{m,H})^{1/4}}$

- Origin?





- Simplest Approach: Electron inertia + MHD

i.e. Ohm's law becomes:

$$\frac{m_e}{e} \frac{d}{dt} \tilde{v}_{\parallel e} + \tilde{E}_{\parallel} = \eta \tilde{J}_{\parallel}$$

Electron inertial effect  $\rightarrow$  electron momentum

- Scale :  $\frac{c}{\omega_{pe}}$

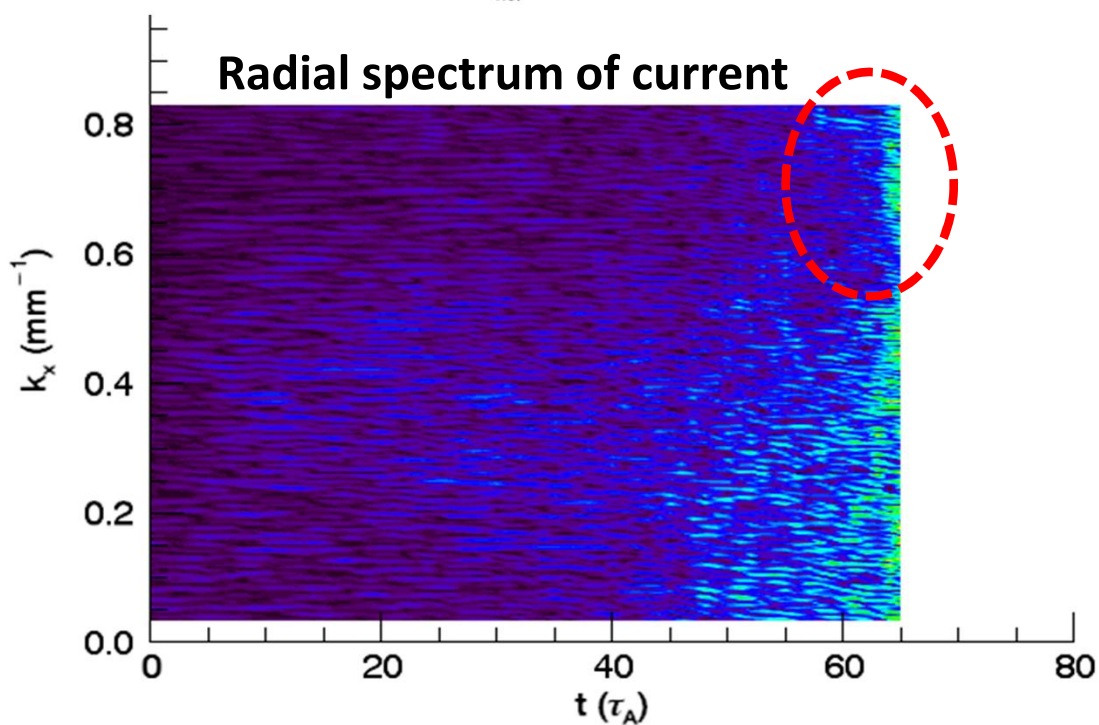
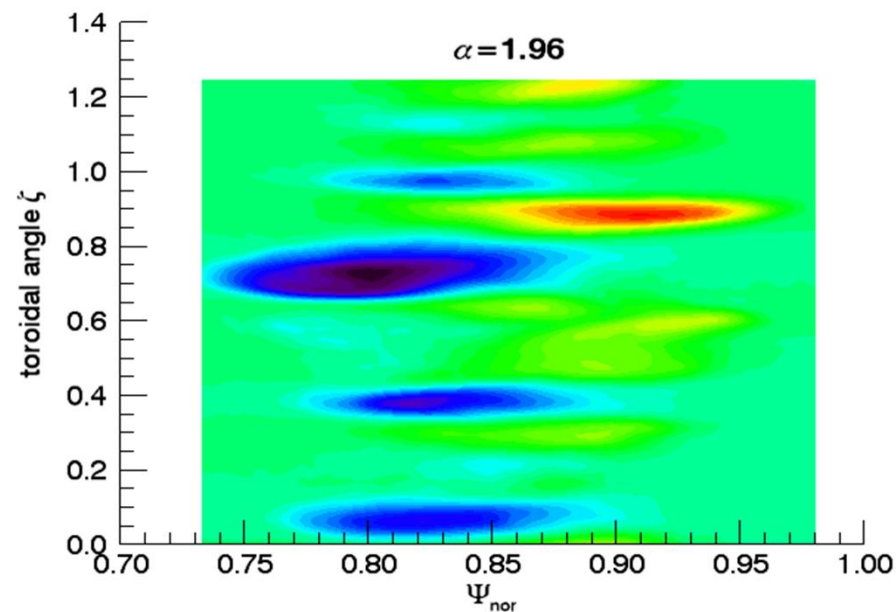
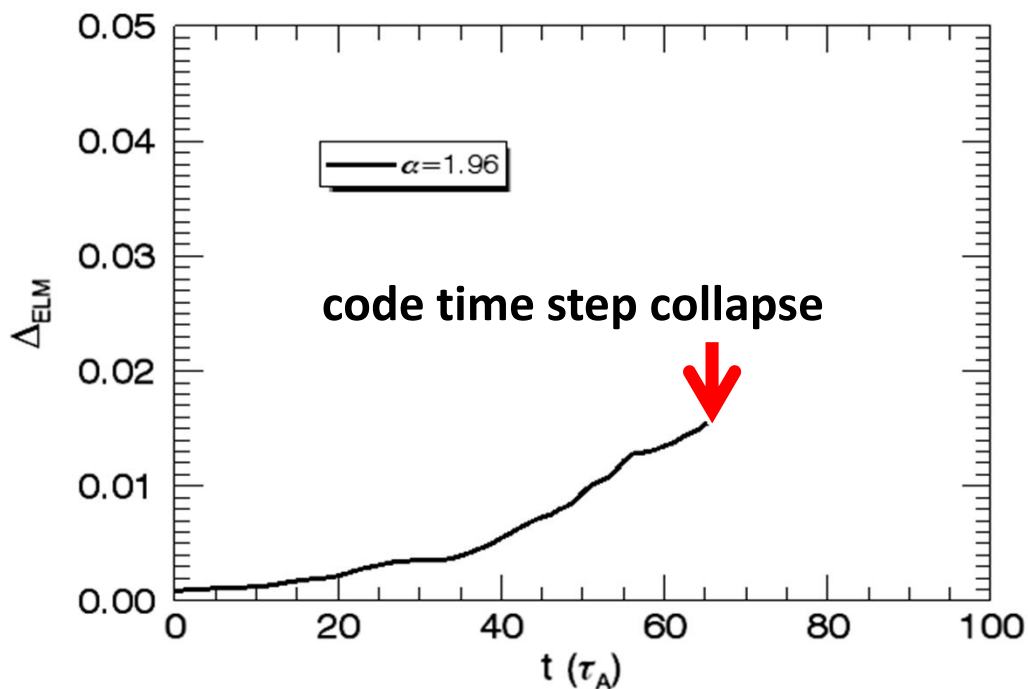
$$\text{Low } n \rightarrow \rho_i \sim c/\omega_{pe}$$

$\rightarrow$  significant effect on linear growth for  $k_{\perp} \left( \frac{c}{\omega_{pe}} \right) \sim O(1)$

P.-B.  $\rightarrow$  'hyper-resistivity' ballooning mode...

- Examine impact on nonlinear evolution...  $\rightarrow$  self-consistent crash?

# Electron inertia and P-B turbulence cannot generate enough current relaxation for ELM crash



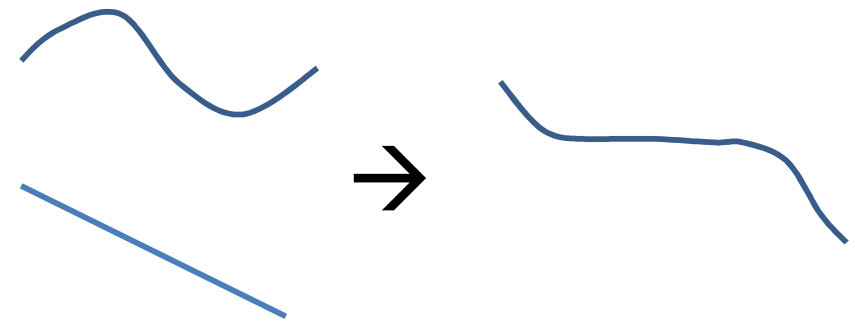
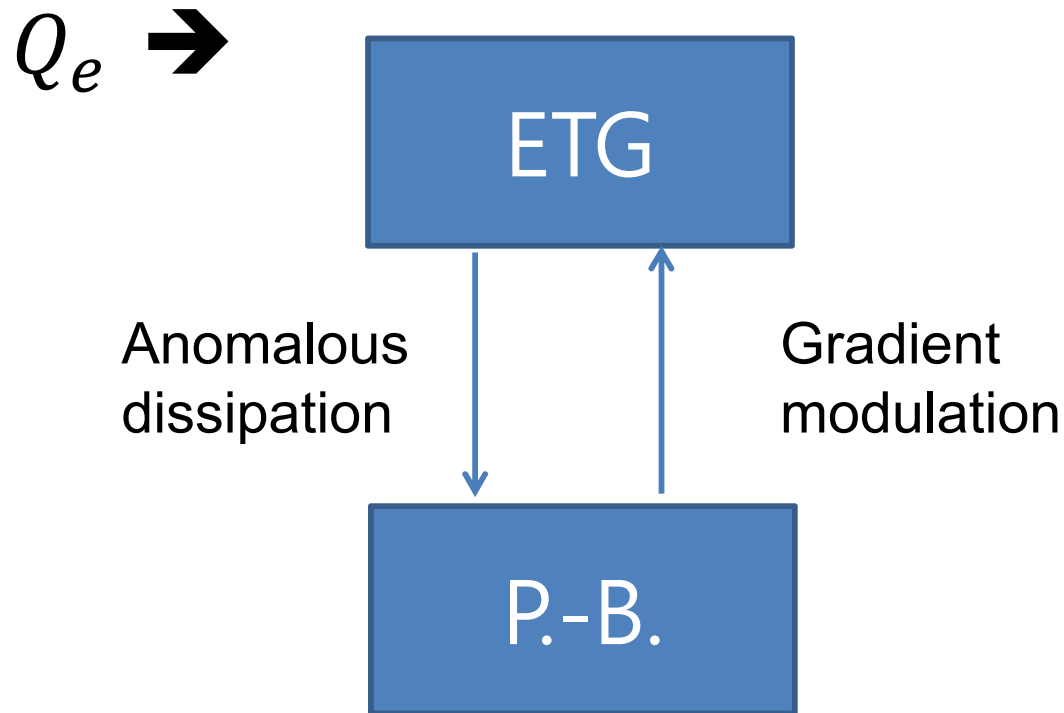
- ✓ Micro-turbulence is needed to generate enough current relaxation
- ✓ The self-consistent nonlinear ELM simulation is a multiple scale issue.

- Interesting candidate for hyper-resistivity
  - ETG turbulence in pedestal?!
- ETG indicated by pedestal micro-stability studies → survive  $\langle V_E \rangle'$
- Mechanism is advection of electron momentum

$$\begin{array}{ccc}
 \text{ITG} & \longleftrightarrow & \text{ETG} \\
 \chi_\phi \sim \chi_i & & \mu_H \sim \chi_e
 \end{array}
 \rightarrow \text{hyper-resistivity linked to pedestal electron heat transport}$$

- $\mu_H \approx \left(\frac{c}{\omega_{pe}}\right)^2 C D_{GBE} \quad D_{GBE} \leftrightarrow L_{Te}^{-1}$ 
  - ↑ anisotropy factor
- Modulation of driving  $\nabla T_e$  by P.-B.'s crucial effect

# Feedback Structure



Approach as disparate – scale modulation problem via gradient evolution due P.-B.

# Partial Summary

- Hyper-resistivity required for dissipation of P.-B. current sheets, and crash
- P.B. + electron inertia insufficient to trigger fast reconnection
- Multi-scale approach to current dissipation is required
- ETG is interesting candidate for origin of  $\mu_H$
- Considerable further work required

### III) Towards a 'Big Picture'

- Is there a 'minimal' model of ELMs?
- What are the key ingredients?
- Might this help us understand ELM-related phenomena better?

→ See: W. Xiao, et al; NF 2013  
T. Rhee, et al; PoP 2012

# Needed: Simple Model...

N.B. full ELM phenomena far beyond “First Principle” Simulations!

- Minimal Model of Pedestal Dynamics
- Necessary Ingredients:
  - Bi-stable flux  $\rightarrow$  capture turbulence, transport, L $\rightarrow$ H transition
  - Fixed ambient diffusion  $\rightarrow$  capture (neoclassical) transport in H-mode pedestal  
N.B. key: how does system actually organize profiles for MHD activity??
  - Hard stability limit  $\rightarrow$  capture MHD constraint on local profile. Can be local. (i.e. ballooning  $\leftrightarrow \nabla P$ ) or integrated (i.e. peeling  $\leftrightarrow J_{BS} \sim \int dr \nabla P \sim P_{ped,top}$ )

N.B. Transport vs 'hard stability'?

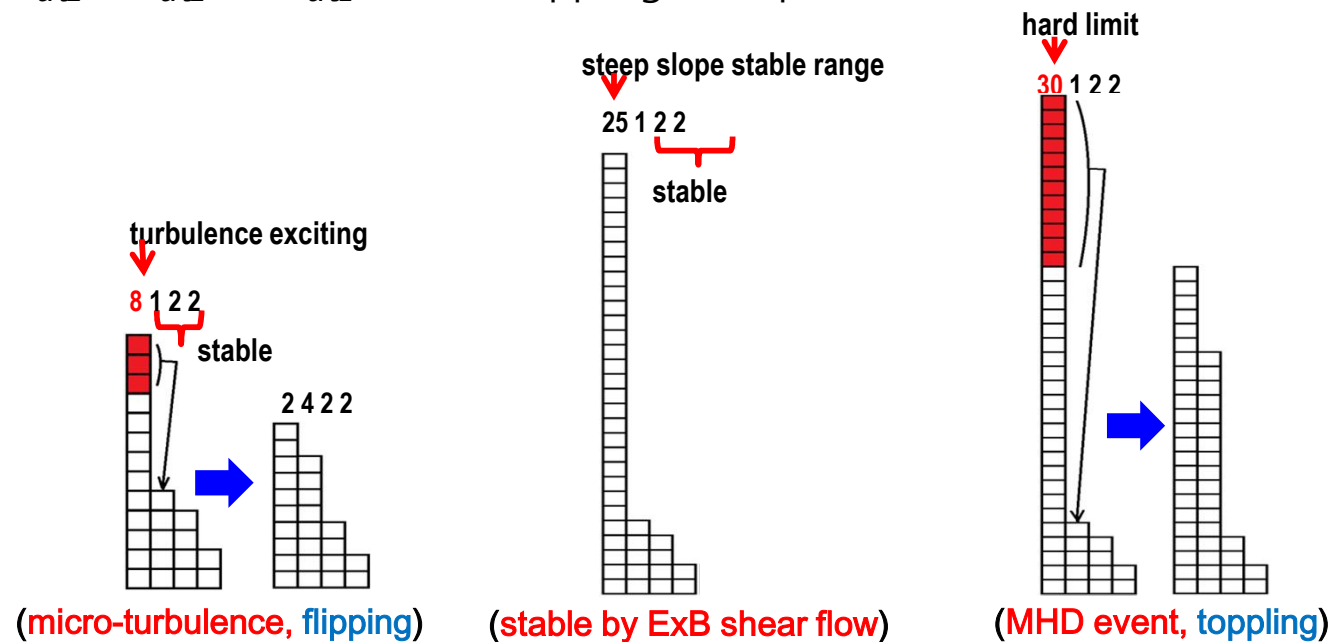
$$\rightarrow Q \sim C \left( \frac{L_{P_{crit}}}{L_P} - 1 \right)^\alpha \quad \therefore c, \alpha \text{ large for 'hard stability limit'}$$

## Sandpile (Cellular Automata) Model

- Toppling rule:  $Z_i - Z_{i+1} > Z_{crit}$  topple  $Y_i$  cells  $\rightarrow$  move adjacent
- Bi-stable toppling:

$Z_i - Z_{i+1} > Z_{crit1} \rightarrow$  toppling, threshold, transport

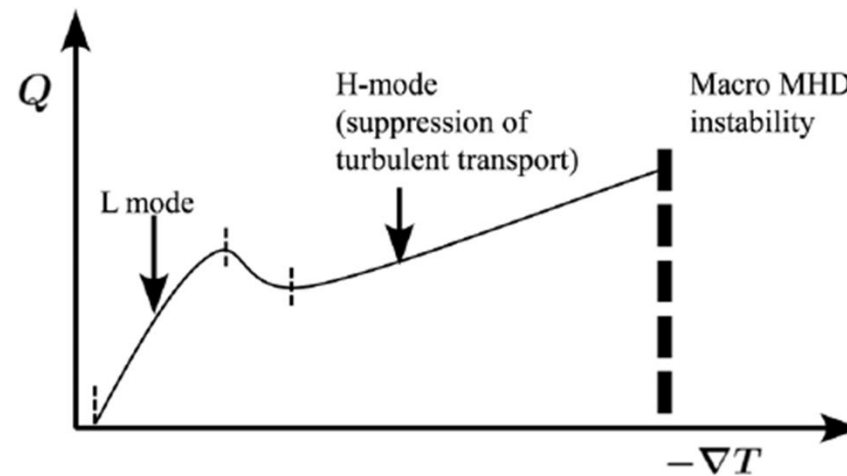
$Z_i - Z_{i+1} > Z_{crit2}$  ,  $Z_{crit2} > Z_{crit1} \rightarrow$  no toppling, transport bifurcation





# Sandpile Model, cont'd:

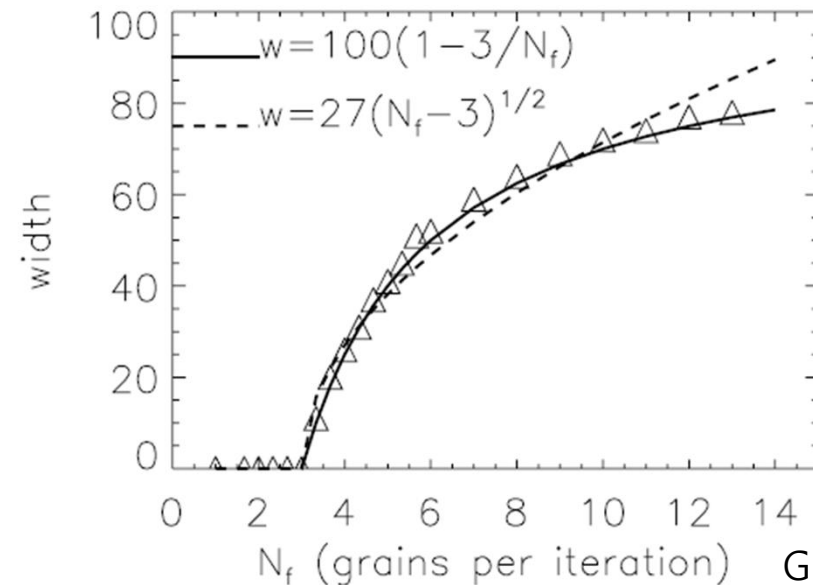
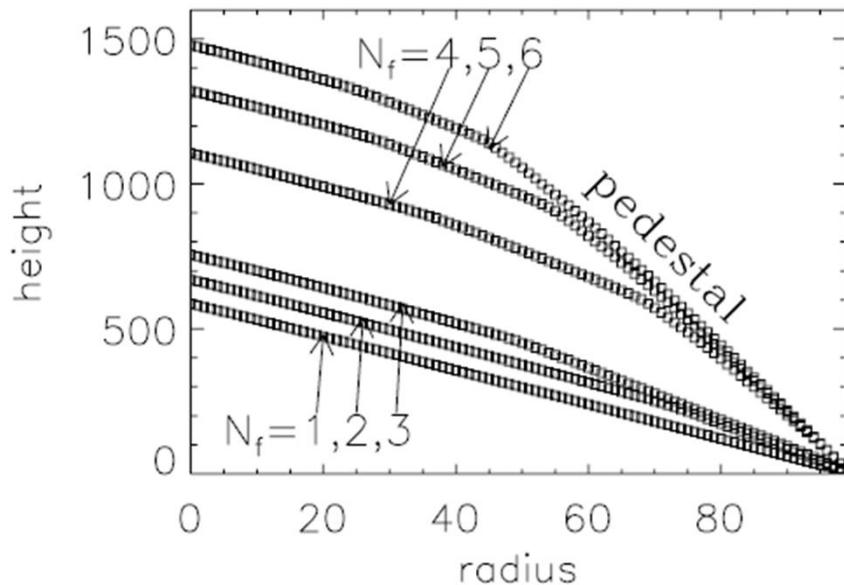
- Constant diffusion  $\rightarrow$  neoclassical transport (discretized)
- N.B. Bi-stable toppling + diffusion  $\rightarrow$  S-curve model of flux



- Hard Limit  $\rightarrow Z_i - Z_{i+1} > Z_{hard} \rightarrow$  topple excess  $Z_i$  according to rule
- Drive:
  - Random grain deposition, throughout
  - Additional "active grain injection" in pedestal, to model SMBI

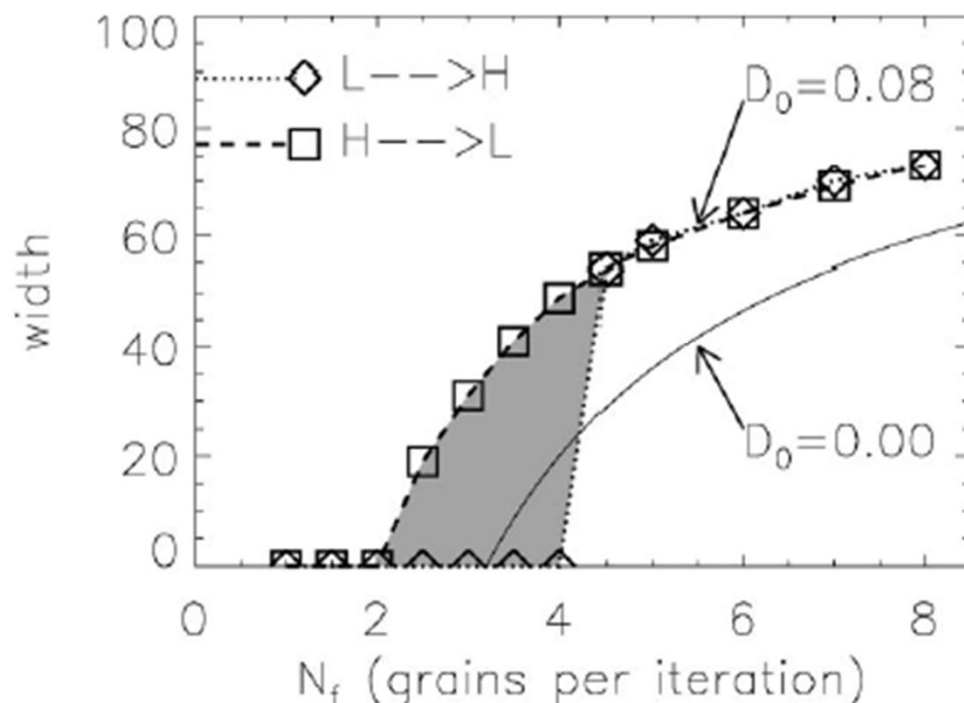
# L → H Transition

- Now try bi-stable toppling rule, i.e. if  $Z_i - Z_{i+1}$  large enough  
 → reduced or no toppling
- Obvious motivation is  $Q = -\frac{\chi \nabla P}{1 + \alpha V_E'^2}$  and  $V_E \approx \frac{c}{eB} \frac{\nabla P}{n}$
- Hard gradient limit imposed
- Transitions happen, pedestal forms!



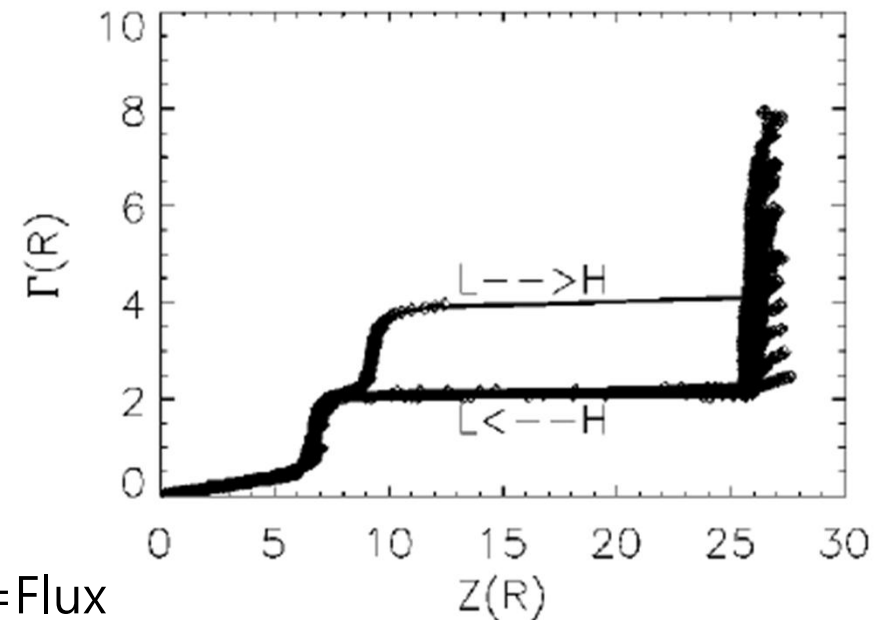
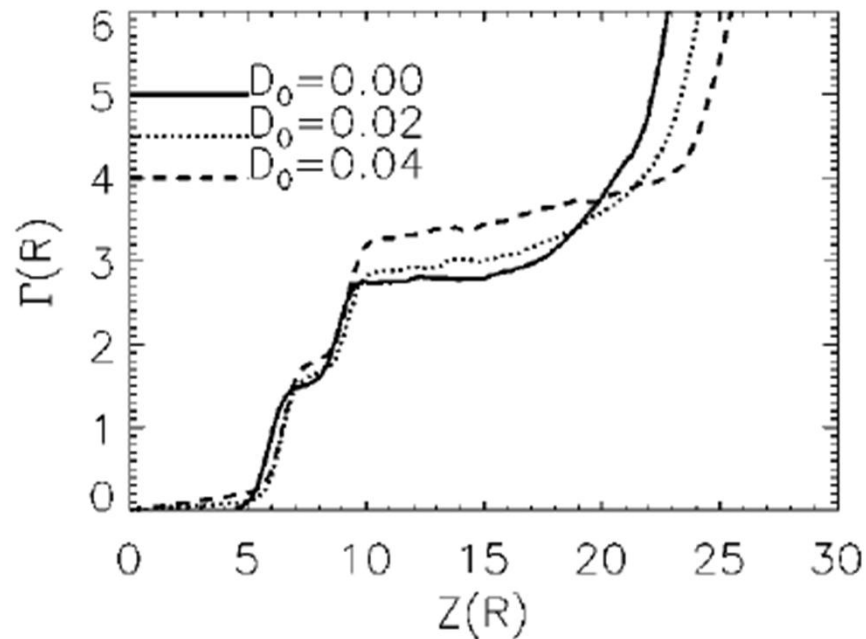
# Note

- Critical deposition level required to form pedestal (“power threshold”)
- Pedestal expands inward with increasing input after transition triggered
- Now, including ambient diffusion (i.e. neoclassical)
  - $N_F$  threshold evident
  - Asymmetry in  $L \rightarrow H$  and  $H \rightarrow L$  depositions



# Hysteresis Happens!

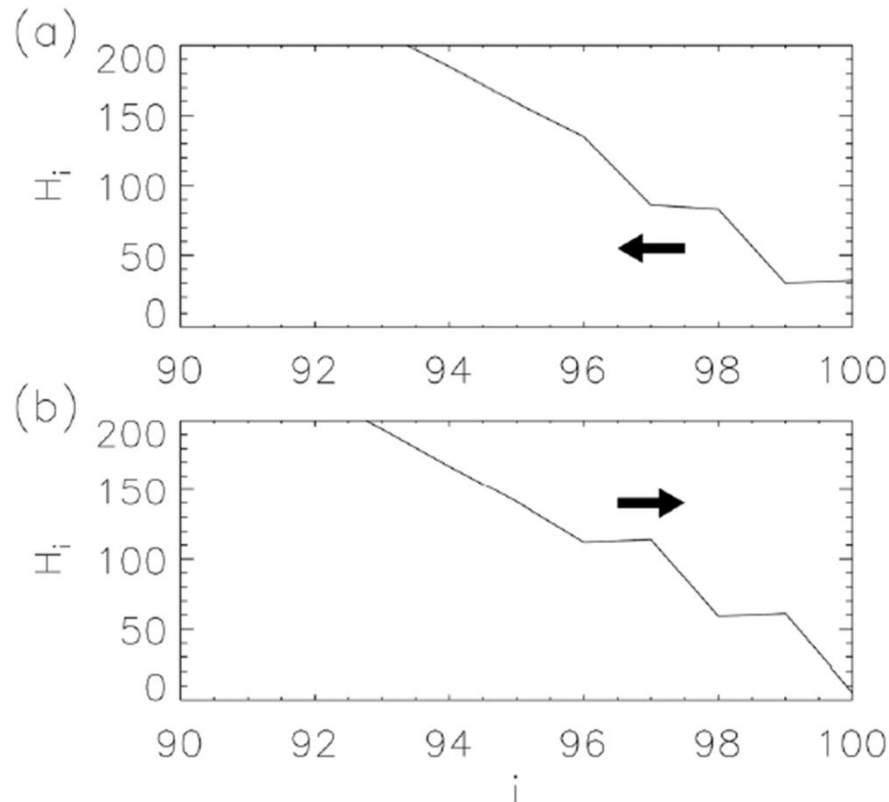
- Hysteresis loop in mean flux-gradient relation appears for  $D_0 \neq 0$
- Hysteresis is consequence of different transport mechanisms at work in “L” and “H” phases
- Diffusion ‘smooths’ pedestal profiles, allowing filling limited ultimately by large events



$\Gamma(R)$  = Flux  
 $Z(R)$  = Mean Slope

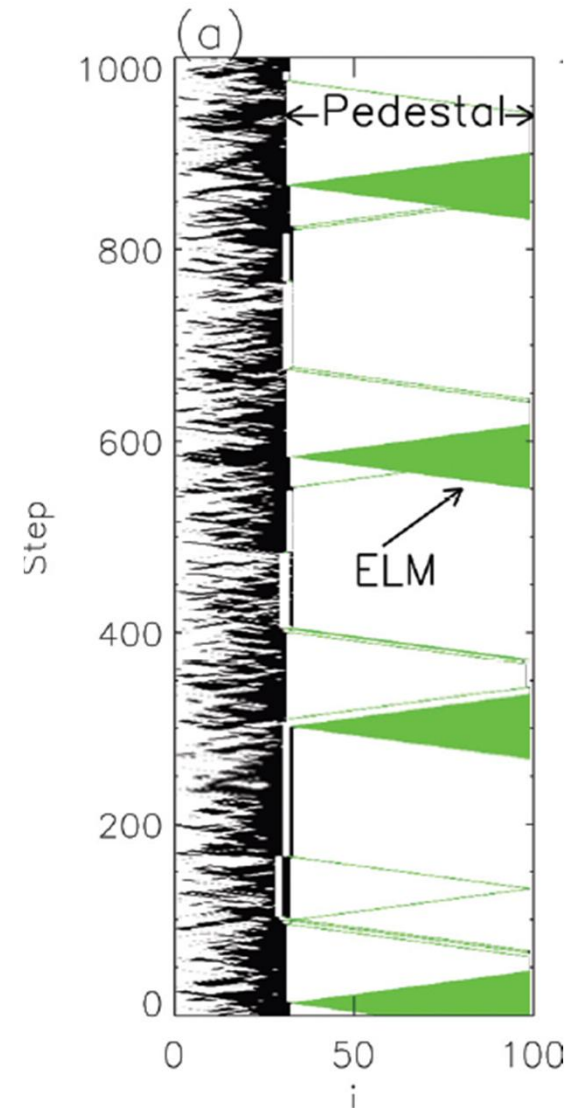
# ELMs and ELM Mitigation

- ELMs happen!
- Quasi-periodic Edge Relaxation Phenomena (ELM) self-organize. Hard limit on  $\nabla Z$  ( $\nabla P$ ) is only MHD 'ingredient' here
- ELM occurs as out  $\rightarrow$  in and in  $\rightarrow$  out toppling cascade



Voids  $\rightarrow$  inward

bump  $\rightarrow$  outward



# ELM Properties

- Periodic with period  $\sim 10^{-2}\tau_p$  .  $\tau_p$  = grain confinement time
- ELM flux  $\sim 500$  diffusive-flux
- ELMs span pedestal
- Period  $\leftrightarrow$  pedestal re-fill (approximate)

## The What and How of ELMs?

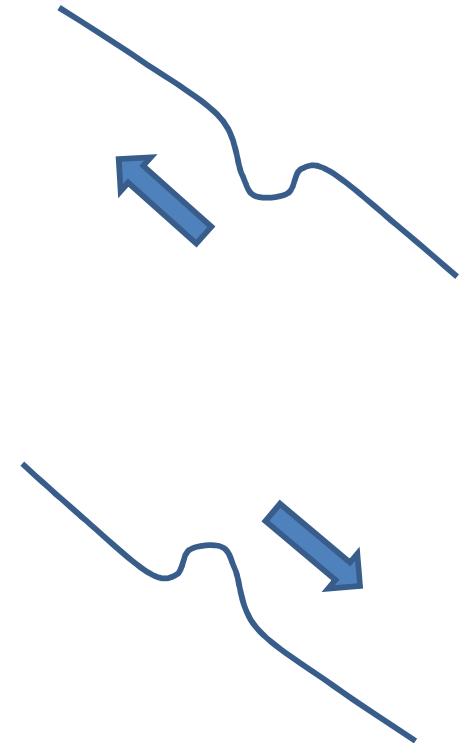
What?

- ELMs are a burst sequence of avalanches, triggered by toppling of 'full pedestal'
- ELMs are not global (coherent) eigen-modes of pedestal

# The What and How of ELMs?

## How?

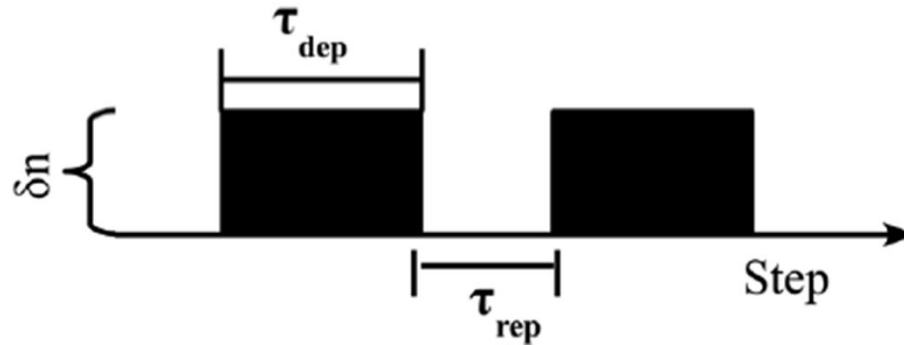
- Toppling cascade:
  - Void forms at boundary, at hard limit
  - Propagates inward, to top of pedestal, triggering avalanche
  - Reflects from top of pedestal, becomes a bump
  - (N.B. core is subcritical  $\rightarrow$  void cannot penetrate)
  - Bump propagates out, causing further avalanching
  - Bump expelled, pedestal refills



N.B. ELM phenomena appear as synergy of H-phase, diffusion, hard limit

## With Active Grain Injection (AGI):

- AGI works by adding a group of grains over a period  $\tau_{dep}$
- Can repeat at  $\tau_{rep}$

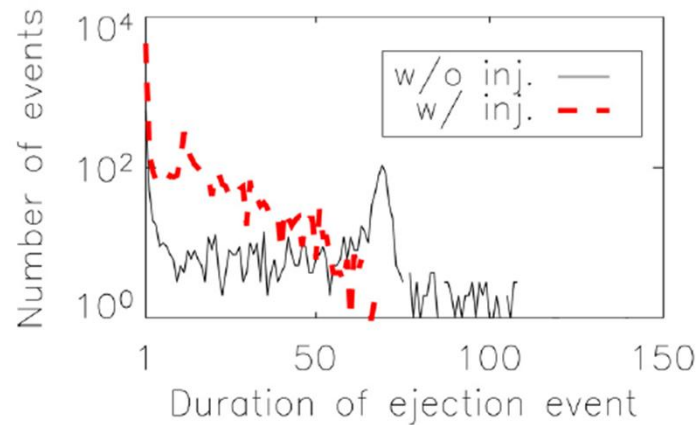


- Obviously, model cannot capture dynamics of actual SMBI, time delay between injection and mitigation. See Z. H. Wang for injection model
- Model can vary strength, duration, location

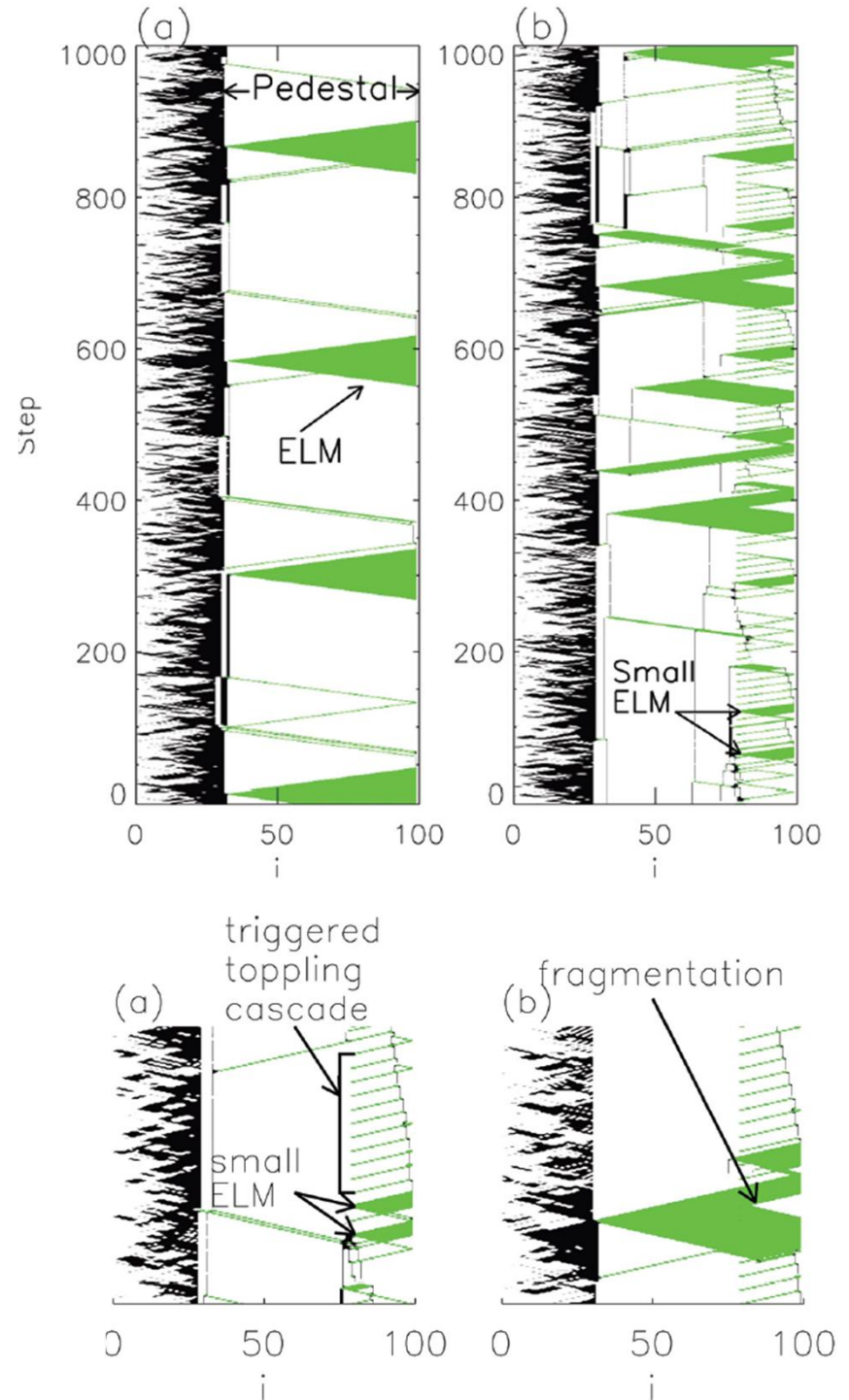


# Results with AGI

- AGI clearly changes avalanche distribution, and thus ELM ejection distribution

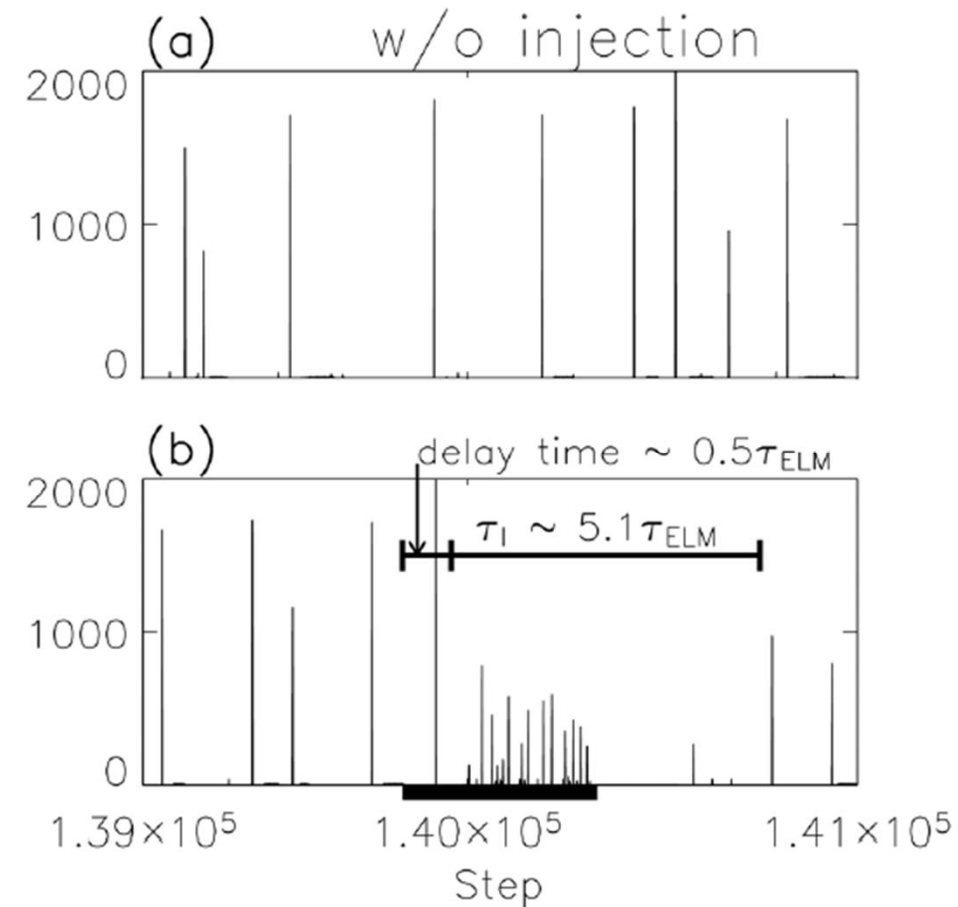


- Mitigation due fragmentation of large avalanches into several smaller ones
- Injection destroys coherency of large avalanches by triggering more numerous small ones
- Consistent with intuition



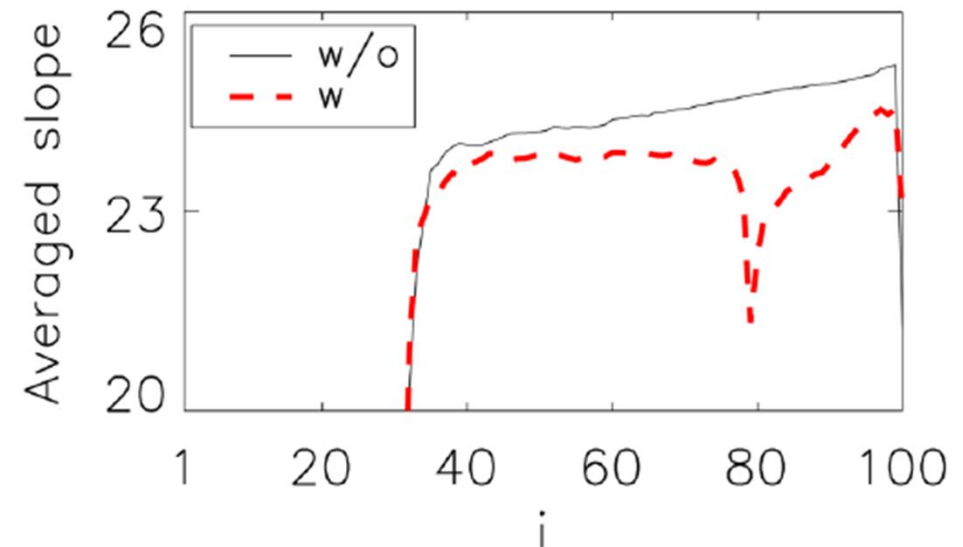
## Edge Flux Evolution (in lieu $D_\alpha$ )

- $A/A_0$  drops,  $f/f_0$  increases
- An “influence time”  $\tau_I$  is evident → duration time of mitigated ELM state
- $\tau_I \sim 5 \tau_{ELM}$



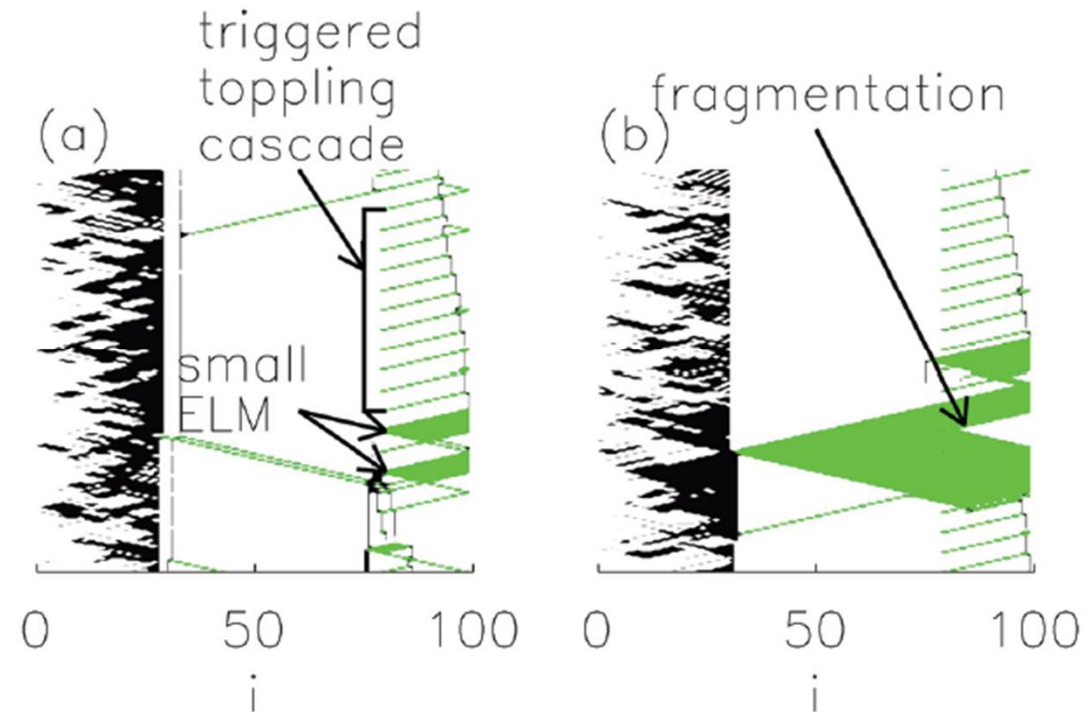
# AGI tends to reduce gradient at deposition region

- Drive triggers local toppling  $\rightarrow$  prevents recovery of local gradient
- 'flat spot' is effective beach, upon which avalanches break
- $\tau_I$  is recovery time of deformed local gradient
- Related to question of optimal deposition location

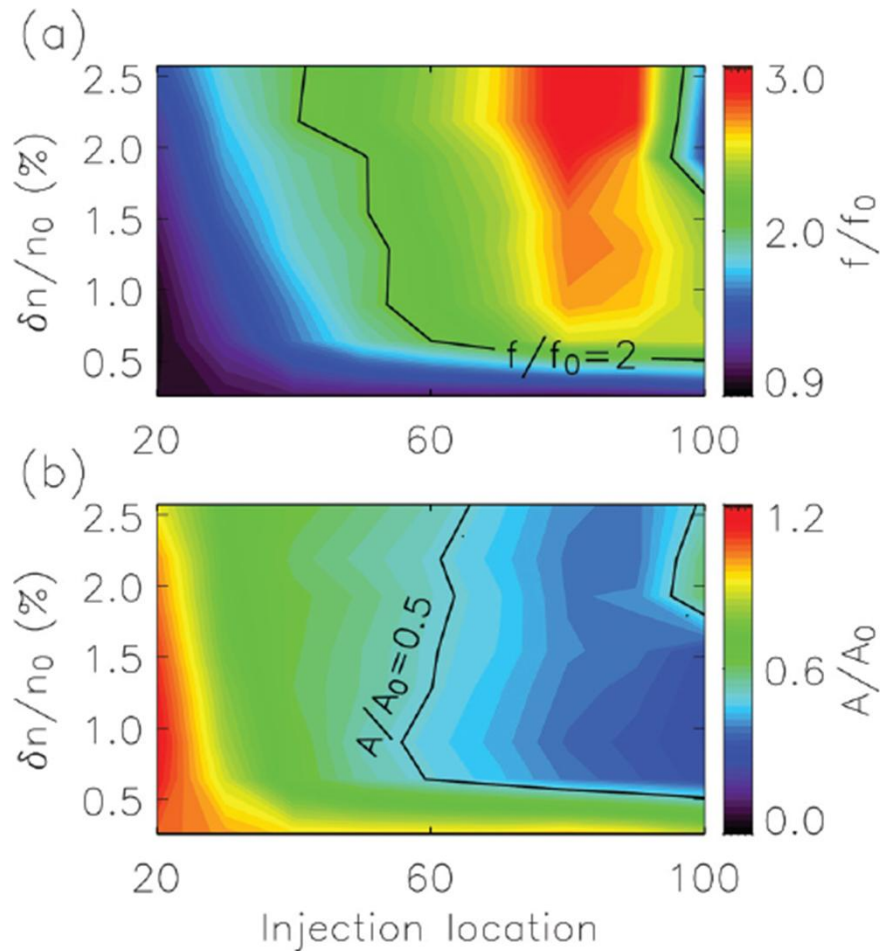


# Which deposition location is optimal?

- Clue: deep deposition, at top of pedestal, allows avalanches to re-establish coherence 'behind' deposition zone
  - Clearly desirable to prevent large avalanches from hitting the boundary
- points toward deposition at base of pedestal as optimal



# Results of Study on Deposition



X  $\rightarrow$  location  
Y  $\rightarrow$  injection intensity

Color: Red high  
Purple low

Results of model study  
point toward optimal  
deposition near pedestal base

- Study suggests optimal location slightly inside pedestal base
- Here  $20 \leq i \leq 100 \rightarrow$  pedestal domain

Here  $\rightarrow$  optimal location  $\sim 80$

# Summary of Reduced Model Results

---

- ELM phenomena emerge from synergy of bi-stable turbulence, ambient diffusion and hard gradient limit. ELM appears as result of avalanche in pedestal
- N.B. Multi-mode interaction necessarily triggers avalanching
- SMBI mitigation may be understood as a consequence of fracturing of pedestal-spanning avalanches

# Conclusions – Coarse Grained

# Conclusions

---

- ELM phenomena are intrinsically multi-mode and involve turbulence
- P.-B. growth regulated by phase correlation
  - determines crash + filament vs turbulence
- Phase coherence can be exploited for ELM mitigation
- Hyper-resistivity dissipation is likely a multi-scale phenomena
- ELMs appear as pedestal avalanching in reduced model



# Where to Next?

- 
- Simulations **MUST** move away from IVP – even if motivated by experiment – and to dynamic profile evolution, with:

- sources, sinks i.e. flux drive essential
- pedestal transport model
- anomalous electron dissipation

i.e. → - what profiles are actually achieved?  
- how evolve near P.-B. marginality?

- Should characterize:
  - pdf of phase fluctuations, correlation time
  - Dependence on  $\tau_c$  control parameters
  - Threshold for burst
- Need understand feedback of P.-B. growth on turbulent hyper-resistivity
- Continue to develop and extend reduced models.